

# *Strict Conditionals.*

## *Replies to Lowe and Tsai*

JAN HEYLEN  
*KU Leuven, Leuven, Belgium*  
LEON HORSTEN  
*University of Konstanz, Konstanz, Germany*

*Both Lowe and Tsai have presented their own versions of the theory that both indicative and subjunctive conditionals are strict conditionals. We critically discuss both versions and we find each version wanting.*

**Keywords:** Strict conditionals; indicative conditionals; subjunctive conditionals.

### 1. *Introduction*

In the vast literature on conditionals there are some theories that give a unified account of both indicative and subjunctive conditionals in natural language — see Bennett (2003: ch. 23) for a discussion of the unified accounts Davis (1979, 1983), Stalnaker (1975, 1984), Ellis (1978, 1984) and Edgington (1995, 2003). Lowe (1983, 1995) and Tsai (2016) have both also proposed a unified theory of conditionals.<sup>1</sup> Whereas Ellis and Stalnaker favour a theory according to which conditionals are ‘variably strict conditionals,’ which are of the form  $\phi \Box \rightarrow \psi$  and which are given a similarity semantics (Stalnaker 1968; Lewis, 1973), Lowe and Tsai favour a theory according to which conditionals are ‘strict conditionals,’ which are of the form  $\phi < \psi$  or, equivalently,  $\Box(\neg\phi \vee \psi)$  (Lewis 1912). Likewise, Daniels and Freeman (1980), Warmbrod (1983), von Fintel (2001) and Gillies (2007) also prefer the analysis in terms of strict con-

<sup>1</sup> Lowe (1979, 1980) defends a unified theory based on the claim that, for counterfactuals belonging to so-called ‘Adams pairs’ (Adams 1970) one can find a future-tense, indicative conditional that is equivalent to it.

ditionals, although they limited their analyses to certain classes of conditionals, namely subjunctive or epistemic conditionals. The discussion between proponents of the strict conditionals analysis and the variably strict conditionals analysis is set against the background of fundamental discussion about the respective roles of semantics and pragmatics. In this short critical reply we will focus only on the work of Lowe and Tsai.

Since both Lowe and Tsai defend that natural language conditionals are a kind of strict conditionals, they have to deal with the so-called ‘paradoxes of strict implication,’ which can best be understood against the background of the so-called ‘paradoxes of material implication.’ Suppose that natural language conditionals of the syntactical form ‘If  $p$ , then  $q$ ’ are material conditionals or ‘implications,’ i.e., they are of the logical form  $p \supset q$ , which is equivalent to:

$$(1) \quad \neg p \vee q$$

Lewis (1912: 524) noted that it would follow that one has to accept ‘If Caesar did not die, then the moon is made of green cheese.’ He also noted that it would follow that one has to accept ‘If the moon is not made of green cheese, then Caesar died’ (Lewis 1912: 527). Lewis (1912: 529) generalizes this by pointing out that the following implications hold:

$$(2) \quad \neg p \supset (p \supset q)$$

$$(3) \quad q \supset (p \supset q)$$

The above are known as paradoxes of material ‘implication.’ Lewis contrasted the material conditionals  $p \supset q$  with strict conditionals or ‘implications’  $p \prec q$  and, correspondingly, extensional disjunctions of the form (1) with intensional disjunctions of the following form:

$$(4) \quad \Box (\neg p \vee q)$$

It can be proved that, if one replaces  $\supset$  by  $\prec$  in (2)–(3), then the resulting formulas are no longer valid. However, Lewis and Langford (1932, 174) noted that there are also paradoxes of strict ‘implication,’ namely:<sup>2</sup>

$$(5) \quad \neg \Diamond p \prec (p \prec q)$$

$$(6) \quad \Box q \prec (p \prec q)$$

We are now ready to turn to Lowe’s and Tsai’s respective theories.

## 2. *Lowe on conditionals*

Lowe wants to have a theory according to which all natural language conditionals are a kind of strict conditionals.<sup>3</sup> However, he does accept that (5) presents a problem. Lowe (1995, 48) offers the following ex-

<sup>2</sup> While Lewis and Langford (1932) prove this for their so-called ‘non-normal’ system of modal logic, they are also theorems in the weakest system of ‘normal’ modal logic.

<sup>3</sup> Lowe (1995: 50) does rule out so-called ‘Dutchman conditionals’, e.g., ‘if that is a Ming vase, then I am a Dutchman.’

ample: ‘If I had bought a ticket, I would have won,’ where it is assumed that it is impossible for me to buy a ticket. Another example is the following: ‘If  $0 = 1$ , then the sun will shine tomorrow’ (Heylen and Horsten 2006: 538). As a putative solution, Lowe (1995: 48) considers the following variation:

$$(7) \quad \Box (\neg p \vee q) \wedge \Diamond p$$

Lowe (1995: 48) offers the following counterexample to (7):

$$(8) \quad \text{If } n \text{ were the greatest natural number,} \\ \text{then there would be a natural number greater than } n.$$

However, as Heylen and Horsten (2006: 539) observe,  $n$  is a free variable. This means that we cannot directly talk about the truth or falsity of the example, but only indirectly, namely via the truth or falsity of its universal closure. Better examples in this respect are the following: ‘If  $0 = 1$  and  $1 = 1$ , then  $1 = 1$ ’ and ‘If Frege Arithmetic is consistent, then Peano Arithmetic is consistent’ (Heylen and Horsten 2006: 542). In both cases there is a logical connection between the antecedent and the consequent, namely an application of the rule of conjunction elimination and (a corollary of) Frege’s Theorem respectively. Lowe (1995: 49) revised his solution by putting forward the following second and final variation:

$$(9) \quad \Box (\neg p \vee q) \wedge (\Diamond p \vee \Box q)$$

While according to Lowe natural language conditionals all have (9) as their logical form, Lowe (1995: 49-51) states that the interpretations of the modal operators  $\Box$  and  $\Diamond$  in (9) can vary. Lowe notes that the modal operators can be given the redundant interpretation, which turns (9) into  $(\neg p \vee q) \wedge (p \vee q)$  or, equivalently,  $q$ . As an example, he points to so-called ‘biscuit conditionals’, e.g. ‘there are biscuits on the sideboard, if you want some.’ Paradigm examples of (present-tense) indicative conditionals involve an epistemic reading of the modal operators, whereas paradigm examples of counterfactuals involve an alethic reading of those modal operators. An epistemic reading of the  $\Box$  operator is ‘it is certain that.’ An alethic reading of the  $\Box$  operator is ‘it is inevitable that.’

Lowe (1995) has been criticized by Heylen and Horsten (2006: 539), who provide the following counterexample:<sup>4</sup>

$$(10) \quad \text{If } 2 = 3, \text{ then } 2 + 1 = 3 + 1.$$

As before, there is a logical connection between the antecedent and the consequent, namely an application of Leibniz’s law and the law of self-identity. Furthermore, Heylen and Horsten (2006: 540-545) claim that there is no propositional condition  $X$  that can be expressed in

<sup>4</sup> Heylen and Horsten (2006: 539) also gave the following counterexample: ‘If I am my father, then my father is my father’s father.’ However, as Lowe (2008: 529) pointed out: according to Macbeath (1982) it is possible that someone is his own father in some time travel scenario’s.

terms of proposition letters  $p, q$ , the modal operator  $\Box$ , and the classical propositional connectives such that  $\Box (\neg p \vee q) \wedge X$  is *exactly* strong enough in the sense that there are no intuitively false conditionals that have that logical form and that there are intuitively true conditionals that lack that logical form. They worked with modal system **S5** in the background, because in **S5** every formula is provably equivalent to a ‘flat’ formula, which does not contain modal operators inside the scope of other modal operators. The latter is important, because they claim that ‘it would be scarcely imaginable that the correct interpretation of conditionals essentially involves nested modalities’ (Heylen and Horsten 2006: 540). Against this, Tsai (2016) claims that a proper unified theory of conditionals involves irreducibly nested modalities. We will return to this in section 3.

In reply, Lowe (2008) formulated a methodological criticism and a substantive criticism. Let us take these in turn.

The methodological criticism was twofold: first, he accused Heylen and Horsten to rely on a very narrow selection of examples and, second, he claimed that their examples are not conditionals that are ordinarily used in everyday conversation (Lowe 2008: 528). But a *tu quoque* response can be given. After all, Lowe (1995: 48) gave only one example against the hypothesis that the logical form of conditionals is captured by (7), and this is a ‘mathematical truism.’ A second response is that the use of conditionals like (10) are more wide-spread in ‘mathematical English.’ There are plenty of examples of conditionals with impossible antecedents and consequents in a textbook on computability and (meta-)logic (Boolos et al. 2007: 38, 40, 97, 126, 132, 134, 154, 160, 192, 223, 227, 228, 271, 284, 303) and in a textbook on algebra (Givant and Halmos 2009: 12, 215, 336, 474). For instance, Givant and Halmos (2009: 215) write the following:

If  $q$  were a strictly smaller upper bound of  $E$  in  $B$ , then  $p - q$  would be a non-zero element of  $B$ , and therefore above a non-zero element  $r$  of  $A$ , by density.

However, it is clear that very often or almost always those kind of conditionals are used in the following type of reasoning: ‘If  $\phi$  were the case, then  $\psi$  would be the case. But  $\psi$  is not the case. Therefore,  $\phi$  is not the case.’ This suggests that the following variation on (10) would have been more in accordance with the above mathematical practice:

$$(11) \quad \text{If } 2 = 3, \text{ then } 2 - 1 = 3 - 1.$$

It is easy to see how such a counterpossible can figure in a proof that leads to  $0 = 1$ , contradicting an axiom of arithmetic and, hence, leading to the conclusion that  $2 \neq 3$ . A third response begins by admitting that ‘mathematical English’ is not colloquial English, although we think that it is a fundamental mistake to draw a sharp distinction between the two. Moreover, dialectically speaking, one is forced to go look for examples from logic or mathematics or metaphysics. Otherwise, Lowe could have claimed that the antecedent is not impossible on a narrow

sense of impossibility or that the consequent is not possibly false on a narrow sense of possibility. For instance, ‘If I had participated as an athlete in the Olympic Games, I would first have passed the Olympic Trials.’ It is open for an objector to claim that the antecedent is not metaphysically impossible.

The substantive criticism starts from the observation that the use of ‘=’ obscures whether one is dealing with an indicative (‘is equal to’) or a subjunctive (‘were equal to’).

Suppose that the conditional is in indicative mode: ‘If 2 is equal to 3, then  $2 + 1$  is equal to’. Then the modal operators have to be interpreted as epistemic operators. Lowe (2008: 529–530) suggests the following reading: ‘ $\square$ ’ means ‘it is certain that’ and ‘ $\diamond$ ’ means ‘it is uncertain that not’. Furthermore, Lowe distinguishes between *real* (un)certainty and *feigned* (un)certainty. While there is real certainty that  $2 \neq 3$ , Lowe suggests that in some context uncertainty that  $2 \neq 3$  may be feigned. In those contexts his theory predicts that ‘If 2 is equal to 3, then  $2 + 1$  is equal to  $3 + 1$ ’ is acceptable after all.

Suppose that the conditional is in subjunctive mode: ‘If 2 were equal to 3, then  $2 + 1$  were equal to  $3 + 1$ ’. Lowe considers a possible world  $w$  in which only the numbers 0, 1 and 2 exist. Moreover, in  $w$  ‘3’ refers to 2, so the antecedent of the conditional is true. Furthermore, in  $w$  the adding-one function is partial: only if the input is 0 or 1 is the output defined (and it the standard outcome). In this world there would be no number corresponding to ‘ $2+1$ ’. Lowe claims that the consequent of the conditional is therefore false in that world.<sup>5</sup> In addition, Lowe (2008, 530) claims that a similar strategy works can be used to show that there is a possible world in which (11) is false.

These last considerations by Lowe lead to radical views in ontology and semantics. One implicit assumption is that natural numbers exist only contingently. Most platonists are not happy with that assumption. Another implicit assumption is that the natural numbers have possible existence independently of other natural numbers. Structuralists disagree with this assumption. So, nominalist structuralism is not an option here. Lowe is also assuming that not all mathematical terms are rigid designators (i.e. terms that designate the same object in all possible worlds in which that object exists and that never designate any other object), while Kripke (1980) illustrated the notion of a rigid designator with the help of arithmetical terms (e.g. ‘the smallest prime’).

Finally, it appears to have escaped Lowe’s notice that his special possible world would also make (8) false. But Lowe had claimed that it is intuitively true. Moreover, he has used the intuitive truth of (8) to argue against (7).

In conclusion, Lowe’s version of the theory that natural language conditionals are strict conditionals fails to convince.

<sup>5</sup> This assumes that an atomic sentence is false at a world if at least one the terms occurring in it does not denote anything in that world.

### 3. *Tsai on strict conditionals*

Tsai (2016: 78) starts with (7), which he labels ‘Default’. He defends the extra condition  $\diamond p$  by reference to the so-called Ramsey Test (Ramsey 1929: 247):  $\diamond p$  expresses that  $p$  is an epistemic possibility, so it is open to add it to the ‘stock of knowledge’. Details aside, what matters most here is that the modal operators in (7) are given an epistemic reading. We should also mention that Tsai gives a formal interpretation of the modal language that does not make use of Kripke models but rather of models in the style of Becker (1952), which Tsai has further developed in earlier work (Tsai 2012). However, we will not go into the details, because Tsai (2012: 107, 112) points out that there is an ‘isomorphism’ between Beckerian ‘hi-worlds’ and a defined ‘sub-hi-world’ relation and Kripkean frames, which contain worlds and an accessibility relation between them.<sup>6</sup>

As we have seen in section 2, Lowe offered (8) as a counterexample to (7). Tsai (2016: 82) agrees that (8) is intuitively true. The solution of Lowe was to accept (9). Tsai (2016: 79) observes that a consequence of (9) is that one has to accept one of the paradoxes of strict implication, namely (6). On this basis, Tsai rejects Lowe’s theory and proposes his own solution.

Tsai (2016: 80) proposes what he labels ‘Unified’:

$$(12) \quad (\neg p \vee q) \text{ or } (\Box(\neg p \vee q) \wedge \diamond p) \text{ or } (\Box\Box(\neg p \vee q) \wedge \neg\diamond p \wedge \diamond\diamond p)$$

The idea is that the logical form of a given natural language conditional is one of the disjuncts of (12). Like Lowe, he accepts that sometimes a natural language conditional can have the logical form of a material implication, and he also agrees that this is a rare case. So, it is mainly about the last two disjuncts. The implicit assumption here is that modal principle 4, namely  $\diamond\diamond\phi \rightarrow \diamond\phi$ , is not valid, because otherwise the third disjunct would be contradictory. This means that according to Tsai the modalities involved are irreducibly nested, contrary to the assumption made by Heylen and Horsten (2006). This also entails that the modal operators in (12) cannot be understood as expressing logical modalities (Burgess 1999), mathematical modalities (Hamkins and Linnebo 2019) or metaphysical modalities (Williamson 2016), which figured prominently in the discussion of Lowe, since adequate systems for those notions all contain at least modal principle 4. Next, Tsai (2012: 80-81) makes a puzzling claim, namely that, if one takes material implica-

<sup>6</sup> Tsai’s claim needs to be qualified slightly: to each Beckerian model there corresponds a Kripkean frame with a *serial* accessibility relation, whereas there are Kripkean frames with a non-serial accessibility relation (i.e. at least one of the worlds does not have access to any world). The reason is that Tsai (2012: 109) stipulates that a hi-world is of the form  $\langle U^0, U^1, \dots \rangle$ , where  $U^0$  is an element of the domain  $D$  of the model and, for each  $i \geq 1$ ,  $U^i$  is an element of  $(P^*)^i(D)$ , with  $P^*(X) = P(X) \setminus \emptyset$ . Given Tsai’s epistemic reading of the modal operator, this qualification is not important, since it is generally accepted that non-serial accessibility relations are inadequate for modelling rational belief and knowledge.

tions out of consideration and if one accepts the validity of  $\Box\phi \rightarrow \phi$  or, equivalently,  $\phi \rightarrow \Diamond\phi$ , (12) can be ‘reduced’ to what he labels ‘Core’:

$$(13) \quad (\Box(\neg p \vee q) \wedge \Diamond p) \text{ or } \wedge (\Box\Box(\neg p \vee q) \wedge \Diamond\Diamond p)$$

However, while (13) logically follows from the last two disjuncts of (12), the converse is not true, even on the assumption that Tsai mentions. In any case, Tsai (2016: 81) labels the second disjunct of (13) ‘Subjunctive’ and he adds that it is relevant when ‘ $p$  is deemed impossible.’ Given that  $\Diamond$  is supposed to be the epistemic possibility operator,  $\Diamond\Diamond$  has to be understood as the *epistemic* possibility of the epistemic possibility.

With the above theory Tsai (2016: 82) tries to account for the intuitive truth of (8). He invites us to imagine a ‘pseudo-mathematical system’ in which there is a greatest natural number  $n$ . This already raises two questions. First, how is imagination related to the epistemic possibility of the epistemic possibility? For a mathematician who has *reflective* knowledge about there not being the largest natural number there is no epistemic possibility that there is the epistemic possibility of  $n$  being the largest natural number. Second, what *are* pseudo-mathematical systems? Perhaps Tsai could take a cue from Lowe and imagine a world in which not all numbers exist and/or in which mathematical vocabulary is interpreted in a non-standard way. But even if these questions can be answered satisfactorily, there is the problem that there is no Beckerian or Kripkean model in which (i) the antecedent of (8) is possibly possible and (ii) (8) is necessarily necessary. The reason is that the consequent, which can be formalized as  $\exists x x > n$ , is logically equivalent to the negation of the antecedent, which can be formalized as  $\neg\exists x x > n$ . Suppose now that there is some possibly possible world at which the antecedent is true.<sup>7</sup> Then by the necessity of the necessity of the material implication the consequent also has to be true at that possibly possible world. Yet, there is no Beckerian or Kripkean world in which logical contradictions are true, even with the countenance of the ontological and semantical views Lowe was willing to resort to. Therefore, (8) has to be false on Tsai’s theory.

For another counterexample to Tsai’s theory, consider first the following conditional:

$$(14) \quad \text{If } 1 = 1, \text{ then } 1 = 1.$$

calls it a ‘truism’ and he uses it to argue against Hitchcock (1998: 25), to whom he attributes the view that the logical form of a conditional is the following:<sup>8</sup>

$$(15) \quad \Box(\neg p \vee q) \wedge (\Diamond p \wedge \Diamond\neg q)$$

<sup>7</sup> By ‘possibly possible’ we mean that it belongs to  $U^2$  (Becker) or that it is accessible from some accessible world (Kripke).

<sup>8</sup> Note that Hitchcock (1998) is really talking about *logical consequence*. He adds the condition that it is possible that the *premises* are true and the condition that it is possible that the *conclusion* is false.

But if Tsai is willing to accept (14), then he should also be willing to accept the following:

(16)  $\text{If } 1 \neq 1, \text{ then } 1 \neq 1.$

Surely, (16) is no less a truism. Note that one does not even need the controversial rule of contraposition but only the observation that the antecedent and the consequent are the same. Yet, there is no Beckerian or Kripkean world in which logical contradictions are true. By the way, we take (16) also to be a counterexample to Lowe's theory of subjunctive conditionals.

Neither Lowe's nor Tsai's version of the theory that natural language conditionals are strict conditionals has withstood critical scrutiny.

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