Strict Conditionals: A Negative Result

Author(s): Jan Heylen and Leon Horsten

Source: *The Philosophical Quarterly (1950-)*, Oct., 2006, Vol. 56, No. 225 (Oct., 2006), pp. 536-549

Published by: Oxford University Press on behalf of the Scots Philosophical Association and the University of St. Andrews

Stable URL: https://www.jstor.org/stable/3840971

The Philosophical Quarterly, Vol. 56, No. 225 October 2006
ISSN 0031–8094

STRICT CONDITIONALS: A NEGATIVE RESULT

BY JAN HEYLEN AND LEON HORSTEN

Jonathan Lowe has argued that a particular variation on C.I. Lewis' notion of strict implication avoids the paradoxes of strict implication. We show that Lowe's notion of implication does not achieve this aim, and offer a general argument to demonstrate that no other variation on Lewis' notion of constantly strict implication describes the logical behaviour of natural-language conditionals in a satisfactory way.

I. INDICATIVE CONDITIONALS AND STRICT IMPLICATION

In reaction to Russell's interpretation of conditional sentences as classical material implications, C.I. Lewis suggested that it would be more appropriate to interpret natural-language conditionals as strict implications. He claimed that the truth-conditions of a sentence of the form 'if p then q' are given by the sentence 'necessarily, not p or q'.[1]

Russell and Lewis were at cross purposes. Russell was mainly interested in the logical meaning of conditional expressions in the restricted context of mathematical proofs, whereas Lewis wanted to express the logical meaning of indicative conditionals in natural language in general. In mathematical proofs, the meaning of conditional assertions can be taken to be expressed by the corresponding material implications. But the so-called paradoxes of material implication to which Lewis drew Russell's attention do show that material implications do not capture the truth-conditions of conditional expressions as they are generally used in daily speech.

Attempts have been made to relegate the paradoxes of material implication to pragmatics. Famously, Grice described the paradoxes of material implication as true assertions which violate some of the conversational implicatures. But the attempts so far made in this direction are generally regarded as unsatisfactory: even neo-Griceans admit this.[2] There exist also

[1] C.I. Lewis, 'Implication and the Algebra of Logic', *Mind*, 21 (1912), pp. 522–31.

[2] See, for instance, S. Levinson, *Presumptive Meanings* (MIT Press, 2000), pp. 208–9. A good source for standard criticism of the Gricean approach to the paradoxes is J. Bennett, *A Philosophical Guide to Conditionals* (Oxford UP, 2003), chs 2–3.

Published by Blackwell Publishing, 9600 Garsington Road, Oxford OX4 2DQ, UK, and 350 Main Street, Malden, MA 02148, USA

non-Gricean attempts to relegate the paradoxes of material implication to pragmatics. One of these is Jackson's, in which the paradoxes of material implication are linked to conventional implicatures. This approach has also met with criticism.[3] Thus it seems worth investigating how far it is possible to describe the logical behaviour of conditionals in purely semantic terms.

In the past, some philosophers have sought to cast doubt on the assumption that natural-language conditionals have truth-values at all. On this view, indicative conditionals can merely be considered more or less acceptable or assertable, but not true or false.[4] These philosophers may well be right, but in this paper we shall assume, along with C.I. Lewis and Russell and a score of contemporary philosophers, that conditionals do have truth-conditions. We shall also assume that all indicative conditionals share a common logical form, even though we recognize that this too is an assumption which could be challenged.

C.I. Lewis' idea of interpreting conditionals as strict implications becomes a determinate proposal against the background of a set of laws governing the notion of necessity and a definite interpretation of the concept of necessity involved. Here Lewis had *logical* necessity in mind. With respect to the laws of necessity, he himself proposed several alternatives, some of which have since become standard systems of propositional modal logic.

It was pointed out early on (by Quine, for instance) that Lewis' theory suffers from a confusion between genuine conditional statements and metalinguistic statements.[5] Sometimes Lewis discusses if–then statements, but elsewhere lapses into talk about statements expressing (logical) implication and consequence. So we shall be explicit here, and state that the theory is intended to reveal the logical form of natural-language statements of the form 'if *p* then *q*'. Also, we restrict ourselves here to indicative conditionals. Specifically, we remain neutral about the question whether counterfactual conditionals have the same truth-conditions as indicative conditionals. Also, we leave out two classes of if–then statements which do not express genuine conditionals, so-called 'biscuit' and 'Dutchman' conditionals. Biscuit conditionals are 'conditional' statements of which the truth-conditions coincide with the truth-conditions of their consequent, for example, 'If you want a glass of wine, there is a bottle in the refrigerator'. Dutchman conditionals are 'conditional' statements of which the truth-conditions are equivalent to

[3] F. Jackson, 'On Assertion and Indicative Conditionals', *Philosophical Review*, 88 (1979), pp. 565–89; cf. Bennett's discussion.

[4] See E. Adams, *A Primer of Probability Logic* (Stanford: CSLI Publications, 1998); D. Edgington, 'On Conditionals', *Mind*, 104 (1995), pp. 235–329.

[5] For a detailed historical account of this matter, see S. Neale, 'On a Milestone of Empiricism', in A. Orenstein and P. Kotatko (eds), *Knowledge, Language and Logic* (Boston: Kluwer, 2000), pp. 237–346, part II.

the truth-conditions of the negation of their antecedents. An example is 'If that's a snake then I'm a Dutchman'. Biscuit and Dutchman conditionals are conditional in name only: they are really categorical assertions of the consequent and denials of the antecedent, respectively.

Since C.I. Lewis' days, it has become clear that if natural-language conditionals are interpreted as strict conditionals, certain odd-sounding judgements concerning the validity of sentences and inferences involving conditional expressions follow. These have become known as the 'paradoxes of strict implication'. For instance, conditionals of the form 'if 0 = 1, then the sun will shine tomorrow' are generally viewed as unassertable, even though the corresponding strict implication is valid in every standard system of modal logic. There seems to be a consensus among philosophers of language that the truth-conditions of a strict implication are *weaker* than the truth-conditions of a natural-language conditional.

II. VARIATIONS

In view of the paradoxes of strict implication, most philosophers have abandoned C.I. Lewis' idea of interpreting ordinary-language conditionals as strict implications. But Jonathan Lowe has rightly observed that this was a hasty conclusion. In two articles, he has tried to amend Lewis' proposal in such a way that the paradoxes of strict implication disappear.[6] The first article is concerned with indicative conditionals, the second primarily with counterfactual conditionals. Lowe believes that counterfactual conditionals have the same *logical form* as subjunctive conditionals. He thinks that in counterfactual conditionals, an alethic modality is implicitly involved, whereas in indicative ones an epistemic modality is involved.[7]

Lowe's proposal[8] is that conditionals of the form 'if p then q' should be logically interpreted as

$$\Box(\neg p \vee q) \wedge (\Diamond p \vee \Box q).$$

Thus a variation on Lewis' proposal is generated. This proposal has the virtue of making false the conditional 'If 0 = 1, then the sun will shine tomorrow'. True, one could achieve this effect in a simpler way, namely, by reading the conditional as

$$\Box(\neg p \vee q) \wedge \Diamond p.$$

[6] E.J. Lowe, 'The Truth about Counterfactuals', *The Philosophical Quarterly*, 45 (1995), pp. 41–59, and 'A Simplification of the Logic of Conditionals', *Notre Dame Journal of Formal Logic*, 24 (1983), pp. 357–66.

[7] 'The Truth about Counterfactuals', pp. 42–3, 49–50.

[8] 'A Simplification', p. 362; 'The Truth about Counterfactuals', p. 49.

But this simple proposal is unsatisfactory. Lowe gives as a counter-example

> If *n* were the greatest natural number, then there would be a natural number greater than *n*.

This is a good counter-example, except that *n* seems to play the role of a free numerical variable. Lowe's example is in the subjunctive mood. In this paper we shall concentrate on indicative conditionals. So we provide a counter-example in the indicative mood:

D. If $V = L$, then there is a Δ^1_2-definable well-ordering of the continuum.

This conditional would come out false even if '$V = L$' were false and the definability claim were true. But we are inclined to regard it as true, for it is a theorem of set theory. So the simpler proposal would generate new paradoxes of strict implication; its reading of conditionals would be too strong. Lowe's more subtle reading of conditionals makes (D) come out true, which is as it should be.

Nevertheless Lowe's variation on Lewis' idea also generates new paradoxical inferences. The reason is that in the context of mathematical proofs, Lowe's proposal is at variance with the judgements generated by Russell's proposal. On the face of it, 'If 2 = 3, then 2 + 1 = 3 + 1' looks like a perfectly correct conditional statement: it is a theorem of arithmetic. But on Lowe's theory, it can never be correctly asserted. In this way the truth-conditions of Lowe's strict conditionals appear more restricted than those of indicative natural-language conditionals. Nothing hinges on our counter-example's being a sentence of mathematics. For instance, on the assumption that it is impossible for one to be one's own father, the sentence 'If I am my father, then my father is my father's father' would do equally well. But the message of Lowe's proposal is that this is not the end of the matter. For lovers of strict conditionals might try other variations on Lewis' idea. As an analysis of sentences of the form 'if *p*, then *q*' they could in principle propose *any* reading

$$\Box(\neg p \lor q) \land X$$

where X is a condition in p, q, $\Box$ and the connectives of classical propositional logic.

It may be worth mentioning that Lowe's strategy has also been applied to the formal treatment of the informal notion of logical entailment: Hitchcock argues that '*p* entails *q*' should be analysed as

$$\Box(\neg p \lor q) \land (\Diamond p \lor \Diamond\neg q).^9$$

[9] D. Hitchcock, 'Does the Traditional Treatment of Enthymemes Rest on a Mistake?', *Argumentation*, 12 (1998), pp. 15–37, at pp. 25–6.

But as we have said, the relation of (logical) implication will be largely left aside in this paper. We focus on indicative conditionals.

III. THE ARGUMENT

We adopt the abstract viewpoint and develop an argument to show that Lowe-like variations on C.I. Lewis' proposal can *never* capture the truth-conditions of natural language indicative conditionals. We shall show that variations on Lewis' strict implication are either *too weak* in some respects, or *too strong*. That is, the reading they provide either classifies certain unacceptable conditional statements as true, or it classifies certain acceptable conditional statements as false.

To this end, we consider the lattice of propositions which can be expressed in terms of p, q, $\Box$ and the classical propositional connectives. This lattice can be partially ordered according to information content, with $\bot$ (Falsum) at the top and $\top$ (True) at the bottom. In algebraic logic, this partial information ordering is known as the Lindenbaum algebra with the order relation reversed (see Fig. 1).

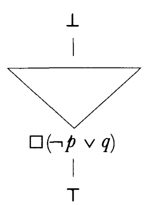

Figure 1: S5 lattice generated by $\{p,q\}$

The size of this lattice is of the order 2^{32}, which at first sight seems a bit discouraging.[10] But we do not have to survey all these propositions. We already know that the reading $\Box(\neg p \vee q)$ is too weak. So we need not consider formulae that are weaker than or incomparable with $\Box(\neg p \vee q)$. We shall consider the propositions which lie just above $\Box(\neg p \vee q)$ in the lattice. We shall argue for all of them that as readings of indicative natural-language conditionals they are either too weak or too strong.

Our main argument rests on two assumptions. The second of these assumptions is only temporary: it will be removed later in the paper.

The first assumption is that the correctness of a reading of conditional statements does not hinge on laws governing *iterations* of modal operators. If the notion of necessity involved is governed by the S5 laws, then iterations of modalities can always be eliminated. But even if the laws governing necessity are significantly weaker, it would scarcely be imaginable that the correct interpretation of conditionals essentially involves nested modalities. The resulting readings would be just too complicated for humans to use in ordinary reasoning. So we shall assume that the putative explications of

[10] See R. Carnap, 'Modalities and Quantification', *Journal of Symbolic Logic*, 11 (1946), pp. 33–64, at p. 48.

indicative conditionals do not contain nested modal operators. Equivalently, we proceed on the assumption that only the S5 lattice is relevant for our investigation.

The second assumption is that X can be taken to be a *purely modal condition*. In other words, all occurrences of the propositional atoms p and q in X are in the scope of $\Box$. For counterfactual conditionals, this assumption would be contentious. For some claim that the correctness of a counterfactual conditional entails the *factual falsity* of its antecedent, and this factual falsity is usually regarded as not being entailed by the modal relation in which p and q stand. But, as was stated at the outset, in this paper we make no claims concerning the logical form of counterfactuals. Concerning indicative conditionals, it seems at least *prima facie* less likely that their logical meaning contains an irreducibly factual component. In any case, this second assumption will ultimately prove to be inessential to our argumentation.

For the rest, our argument makes use of only very weak modal assumptions. Specifically, the basic and uncontroversial normal modal logic T suffices for our argument, to which we now turn.

We shall be looking at side-conditions X which increase the information content of, and therefore are not entailed by, $\Box(\neg p \vee q)$. The proposition expressed by such an X can be put in disjunctive normal form. In general, $\Diamond(\neg p \vee q)$ should not be one of the disjuncts, for then the resulting X is implied by $\Box(\neg p \vee q)$. So we turn to the slightly weaker

$$\Diamond(p \vee q) \vee \Diamond(p \vee \neg q) \vee \Diamond(\neg p \vee \neg q).$$

This value for X is equivalent to $\neg\Box\bot$. Therefore conjoining it does not strengthen the content of $\Box(\neg p \vee q)$. The same holds for disjunctions of two of the disjuncts of the preceding formula, such as $\Diamond(p \vee q) \vee \Diamond(p \vee \neg q)$. They too do not increase the strength of $\Box(\neg p \vee q)$. So we *minimally* increase the strength, and look at the following putative values for X:

1. $\Diamond(p \vee q)$
2. $\Diamond(p \vee \neg q)$
3. $\Diamond(\neg p \vee \neg q)$.

It can be shown that each of these results in a reading of conditionals which is too strong. We first consider the reading $\Box(\neg p \vee q) \wedge (1)$. The sentence 'If $2 = 3$, then $2 + 1 = 3 + 1$' is true. But if it is interpreted in accordance with $\Box(\neg p \vee q) \wedge (1)$, it is judged false. The following theorem of number theory offers a less childish sort of example:

G. If Goldbach's conjecture is true, then every number greater than 17 is the sum of three distinct primes.

It has been proved that this conditional statement can be strengthened to an equivalence. Now suppose that contrary to expectation, Goldbach's conjecture turns out to be false. Then the consequent of the conditional statement (G) would also be false, and our reading $\Box(\neg p \vee q) \wedge (1)$ would judge the conditional incorrect. But surely (G) would still be true. Granted, once it is known that Goldbach's conjecture is false, it would be more appropriate to assert the conditional connection in a counterfactual manner. But that is no more than an application of the Gricean maxim of informativeness. The corresponding counterfactual would convey that the antecedent is known to be false. The indicative conditional (G) would still be correct – we would not have to rewrite the mathematics textbooks. But it would be pragmatically defective: our assertion would not be maximally informative.

Next, we consider the reading $\Box(\neg p \vee q) \wedge (2)$. Here too we find a simple counter-example

If $0 = 1$ and $1 = 1$, then $1 = 1$.

Again this sentence seems true if it has any truth-value at all. Yet on the reading under consideration, it comes out false. Lest this example is also considered on the simplistic side, the following assertion makes the same point:

F. If Frege Arithmetic is consistent, then Peano Arithmetic is consistent.

Russell taught us that Frege Arithmetic is inconsistent. The antecedent of this conditional assertion therefore is necessarily false, whereas the consequent is necessarily true. So this conditional does not satisfy $\Box(\neg p \vee q) \wedge (2)$. But the conditional statement (F) is generally regarded as correct, for Frege's deduction of the Peano axioms from second-order logic with the unrestricted abstraction axiom was flawless.

Finally, we look at $\Box(\neg p \vee q) \wedge (3)$. This proposal can be countered with

If $2 = 2$ then $2 + 1 = 2 + 1$

which is true. After all, it is a theorem of arithmetic. But the proposed reading makes it false. So we conclude that all these readings are too strong. As before, nothing hinges on the counter-examples' being mathematical statements. Non-mathematical statements serving the same purpose are readily found, as the reader can check.

We also have to consider the following putative values for X:

4. $\Box\neg p \vee \Diamond q$
5. $\Diamond\neg p \vee \Box q$.

To both of these, the following sentence serves as a counter-example:

If I am my father, then Belgium is a dictatorship.

This conditional does not ring true. But according to the two readings under consideration, they are true. Therefore the readings generated by these two putative values of X are too weak. So again we crawl incrementally up the lattice, and encounter as a putative value of X the formula

6. $\Box\neg p \vee \Box q$.

But this value yields a logical interpretation of conditional statements which is too strong. This is displayed by

If I have 3 euros in my pocket, then I have more than 2 euros on me.

This seems a true conditional statement. But both its antecedent and its consequent express contingent propositions. So the logical reading under consideration makes it false.

Modal side-conditions X in only one variable (p or q) need not be considered, for they are all stronger than the minimal side-conditions which we have considered and found too strong. For similar reasons, side-conditions in which complex propositional formulae occur in the scope of a modal operator need not be considered. They either are entailed by $\Box(\neg p \vee q)$ and therefore carry no extra information, or entail a modal side-condition in which only proposition letters or negations thereof occur in the scope of a modal operator, and this has been found to be too strong. As an example, the side-condition $\Box[(p \wedge q) \vee (\neg p \wedge \neg q)]$ is stronger than $\Diamond\neg p \vee \Diamond q$, yet weaker than $\Box\neg p \vee \Box q$. So should we not consider it? No, for it is stronger than $\Diamond p \vee \Diamond\neg q$, which has been shown to be already too strong.

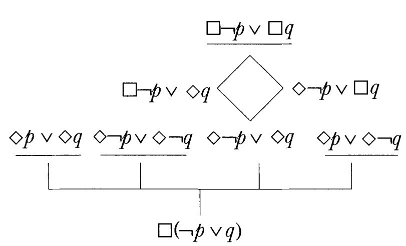

Figure 2: Capping 1

Summarizing, the situation can be described thus. Just above $\Box(\neg p \vee q)$ in the lattice, we find readings which are either too strong, (1)–(3), or still too weak, (4)–(5). But just above these readings which are still too weak, we find one single reading (6) which is again too strong. So we have 'capped' the reading $\Box(\neg p \vee q)$ by purely modal readings which are either too weak or too strong. (See Fig. 2; values for X that yield readings which are too strong are underlined.)

Now we shall show that the second assumption of our argument is not essential. We shall reconsider the readings that are too weak, and show that conjoining even the weakest extra factual condition to any of them results in a reading which is too strong.

First, we consider C.I. Lewis' basic reading $\Box(\neg p \vee q)$. The weakest factual conditions that can be conjunctively added to this reading are $p \vee q$, $p \vee \neg q$, $\neg p \vee q$, $\neg p \vee \neg q$. Conjoining $p \vee \neg q$ yields a reading which is too strong. For a counter-example,

> If we are brains in a vat, then the outside world exists

seems to be a correct assertion, for if we are brains in a vat, then at least the vat must exist. Yet it seems eminently plausible that the antecedent is false and the consequent is true. So the conditional statement does not satisfy the side-condition, and therefore it is wrongly classified as false by our reading.

We now consider $p \vee q$ and $\neg p \vee \neg q$. For $p \vee q$, the assertion

> If there are seven planets in our solar system, then the number of planets is prime

might be uttered by someone in the eighteenth century, after the discovery of Uranus, but before the discovery in the nineteenth century of Neptune. And for $\neg p \vee \neg q$, the assertion

> If the earth revolves around the sun, then the nearer of the fixed stars should appear to move relative to the farther ones

might be uttered by a sixteenth-century astronomer. These readings are judged incorrect by the respective readings. Yet they appear perfectly sound conditional assertions. This leaves us with $\neg p \vee q$. Conjoining $\neg p \vee q$ adds no information. (This is one of the places in our argument where the T-axiom is used.) For it is this principle which implies that the logical content of $\neg p \vee q$ is weaker than that of $\Box(\neg p \vee q)$. So we contemplate strengthening by conjoining a slightly stronger factual condition: $\neg p$, or q. But each of these is stronger than conditions which have already been shown to yield readings which are too strong. Now we have surveyed all factual conditions just above $\Box(\neg p \vee q)$ (see Fig. 3).

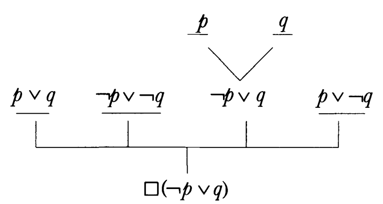

Figure 3: Capping 2

Secondly, we consider the readings (4) and (5). The weakest factual conditions which can be conjunctively added to these readings again are $p \vee q$, $p \vee \neg q$, $\neg p \vee q$, $\neg p \vee \neg q$. $p \vee \neg q$ yields a reading which is too strong. The earlier counter-example (F) about Frege Arithmetic again illustrates this. As for $p \vee q$ and $\neg p \vee \neg q$, we first contemplate adding $p \vee q$ as an extra conjunct of X. The resulting modifications (4a) and (5a) of (4) and (5) are too strong, as is shown by

If I am my father, then my father's first name is the same as mine.

This is an acceptable conditional with an impossible antecedent and a contingently false consequent. Yet according to (4a), this conditional is false. (5a) too would classify this conditional statement as false. So readings (4a) and (5a) are too strong. Adding $\neg p \vee \neg q$ as extra conjunct of X results in readings (4b) and (5b). A counter-example is the statement 'If $3 = 3$, then $3 + 1 = 3 + 1$'. Again this is a correct conditional statement, but it is classified by (4b) and (5b) as false. So these readings are also too strong. Now all we are left with is $\neg p \vee q$, which is too weak. So we must again contemplate adding something slightly stronger to (4) and (5), namely, $\neg p$ or q. But these conditions are stronger than the conditions $\neg p \vee \neg q$ and $p \vee q$, respectively, which have been shown to yield readings that are too strong. Thus the argument is concluded.

We now see that it is no accident that Lowe's proposed variation on C.I. Lewis' idea does not work. The extra condition X which lifts the reading $\Box(\neg p \vee q)$ to an interpretation of conditionals which is exactly strong enough cannot be expressed in terms of p, q, $\Box$ and the connectives of classical propositional logic.

IV. CLASSES OF INDICATIVE CONDITIONALS

The outcome of the argument is on the whole not unexpected. Most philosophers of language today would regard it as unlikely that a variation on C.I. Lewis' strict implication can accurately describe the logical behaviour of indicative conditional statements. This is witnessed by the fact that many of the contemporary philosophical theories about indicative conditionals fall outside the scope of our negative result. Nevertheless, the results of this paper do affect *some* recent theories of indicative conditionals, such as that of Lowe, but also that of Warmbrōd.[11] The upshot of this paper is that most philosophers of language and philosophical logicians *rightly* believe that in order to arrive at the correct logical interpretation of indicative conditionals, a new idea is needed. And this involves challenging either some of our judgements concerning the truth-values of our examples, or one or more of the presuppositions of our negative result.

[11] K. Warmbrōd, 'Epistemic Conditionals', *Pacific Philosophical Quarterly*, 64 (1983), pp. 249–65.

A first option is, as noted earlier, to challenge some of the judgements concerning conditionals which were adduced to refute proposed logical interpretations of indicative conditionals. Lowe himself, for instance, questions whether 'If $2 = 3$, then $2 + 1 = 3 + 1$' has a truth-value. He believes that this sentence is assertable so long as it is simply regarded as an instantiation of the true universal generalization 'For all natural numbers m and n, if $m = n$ then $m + 1 = n + 1$'; but taken as an assertion specifically about the numbers 2 and 3, it is highly paradoxical and has no truth-value. For if $2 = 3$, then arithmetic as we know it is a complete mistake, so the laws of addition cannot be trusted.[12] But this seems hard to maintain. For it amounts to denying the validity of the rule of universal instantiation, which is valid even in partial logic. It must be admitted that many philosophers of language today deny that conditional statements with an impossible antecedent can ever have a truth-value. For some such conditionals, this may appeal ('If $0 = 1$, then it will rain tomorrow'). But for a sentence such as 'If $2 = 3$, then $2 + 1 = 3 + 1$', the immediate appeal of claiming that it has no truth-value seems limited. And its appeal seems to diminish further if the example is replaced by a less elementary one, such as the example of Goldbach's conjecture discussed earlier.

A second option that our argument does leave open is to explicate the logical behaviour of conditionals not just in terms of the propositional variables, the usual logical connectives of propositional logic and the modal operators $\Box$ and $\Diamond$. One can, for instance, introduce a comparative possibility operator $<$ ('It is more possible that ... than that ...'). This is exactly what David Lewis did when he introduced the notion of a variably strict conditional as the logical form of counterfactual conditionals.[13] With the help of a comparative possibility operator one can define a counterfactual operator $\Box\!\!\rightarrow$ as follows:

$$p \Box\!\!\rightarrow q =_{df} \neg\Diamond p \vee [(p \wedge q) < (p \wedge \neg q)]$$

So the idea is that 'if p were the case, q would have been the case' is (non-vacuously) true if and only if it is more possible that p and q are both true than that p is true but q is not. Lewis himself applied the notion only to counterfactual conditionals, but some authors, such as Gillies, have argued that it may also be applied to indicative conditionals.[14] At any rate, variably strict conditionals for the most part fall outside the scope of our argument,

[12] This reply was given to us by Lowe in personal communication.

[13] D. Lewis, *Counterfactuals* (Harvard UP, 1973), pp. 52–6. See also his 'Counterfactuals and Comparative Possibility', repr. in his *Philosophical Papers*, Vol. II (Oxford UP, 1987), pp. 10–11.

[14] A. Gillies, 'Epistemic Conditionals and Conditional Epistemics', *Noûs*, 38 (2004), pp. 585–616.

because variably strict conditionals cannot be defined in terms of the propositional variables, the usual logical connectives of propositional logic and the common modal operators.

A counterfactual conditional is vacuously true whenever it has an impossible antecedent. David Lewis was well aware that paradoxes emerge at this point.[15] He notes that some counterfactual conditionals with impossible antecedents are unassertable, for example,

> If there were a largest prime, pigs would have wings.

It occurred to Lewis that one possible reply would be to install a condition ensuring that the antecedent of a counterfactual conditional would always be possible. His proposal was to introduce a new counterfactual operator with the following contextual definition:

$$p \,\Box\!\!\Rightarrow q \;=_{df}\; (p \wedge q) < (p \wedge \neg q).$$

It is easily derivable that p should be possible whenever $p \,\Box\!\!\Rightarrow q$ is true. But Lewis also pointed out that certain counterfactual conditionals with impossible antecedents seem to be true, for example,

> If there were a decision procedure for logic, there would be one for the halting problem.

A question arises analogous to the question we have considered in this paper, namely, whether there is any variation on the notion of a variably strict conditional such that all paradoxes can be avoided.

At this point one may wonder whether it is possible to extend our argument so as to cover variably strict conditionals too. After all, constantly strict conditionals are just a special kind of variably strict conditional. Indeed, whenever a constantly strict conditional is true, the corresponding variably strict conditional is true. Moreover, we have just shown that the notion of variably strict conditionals also gives rise to paradoxes, and that at least one variation on that notion has been proposed to escape from the paradoxes. It would not be an easy task, however, to extend our argument to variably strict conditionals. Our capping procedure is not suited for this task. One should nevertheless not conclude that our judgements regarding the paradoxes are necessarily better served by a logic of variably strict conditionals.

Thirdly, some maintain that the logical interpretation of indicative conditionals is indeed given by a necessary implication reading, but hold that *contextual factors* influence the interpretation of the notion of necessity

[15] *Counterfactuals*, pp. 24–6; 'Counterfactuals and Comparative Possibility', pp. 18–19.

involved.[16] To put it schematically, the idea is that natural-language conditionals are on the semantic level constantly strict conditionals, whereas on the pragmatic level they are variably strict conditionals.

As long as only *possible* worlds are involved in the interpretation of the notion of necessity, this will be of no help in avoiding the conclusion of the preceding argument. The laws of modal logic appealed to still go through. For if only possible worlds are involved, even when the set of possible worlds is restricted, sentences necessarily true on the context-insensitive reading will still be necessarily true on the contextual reading, and sentences impossible on the context-insensitive reading remain impossible after contextual relativization. Of course, in *some* contexts, contingently true (false) modal sentences become false (true) by contextual restriction of the set of possible worlds. But so long as for each of the examples adduced at least *one* context can be construed for which the evaluation given of that example is correct, the argument goes through. And we maintain that this is the case for the examples that we have given. That Lowe's theory only involves possible worlds can be gathered from the fact that his logic of necessity is closed under the necessitation rule.[17] The same is true for Warmbrōd's theory. Even though the latter does not provide an axiomatization of the modal logic behind his proposal, he asserts that it must be closed under necessitation.[18]

A more thoroughgoing contextualist theory might involve *impossible worlds*. These might be invoked in order to represent contextually determined situations in which impossible states of affairs such as the falsehood of certain logical or mathematical facts are true. According to such theories, the modal operator will not in each context be governed by an extension of the normal modal logic T. On such readings of the modal operator, the truth-value of some of the examples adduced in the previous section in the light of certain variations of C.I. Lewis' strict implication reading may indeed change. This would mean that the argument no longer goes through: we have arrived at a pragmatic escape from the conclusion of the argument.[19]

Fourthly, one could in a Gricean style view as *conversational implicatures* the extra conditions X conjoined to the strict implication reading of indicative conditionals, and try to explain away some of our counter-examples by

[16] This idea is clearly described in K. von Fintel, 'Counterfactuals in a Dynamic Context', in M. Kenstowicz (ed.), *Ken Hale: a Life in Language* (MIT Press, 2001), pp. 123–52, at p. 130. Lowe, in 'A Simplification of the Logic of Conditionals', p. 357, asserts that this phenomenon applies to indicative conditionals, and in 'The Truth about Counterfactuals', p. 55, that it is also the case for counterfactuals. It is a key component of the theory of Warmbrōd.

[17] Lowe, 'A Simplification of the Logic of Conditionals', p. 360.

[18] Warmbrōd, 'Epistemic Conditionals', p. 265, n. 21.

[19] Thanks to an anonymous referee for pointing this out.

finding reasons why the expected implicatures do not apply. It is not clear to us at the moment how promising such an approach would be.

A fifth option would be to drop the assumption that a common logical form is shared by all indicative conditionals, but maintain that a natural subclass of indicative conditionals is governed by a strict implication. This might involve isolating a subclass of indicative conditional assertions as expressing *inferential* conditionals, and arguing that the logical behaviour of these conditionals is accurately described by Lewis' strict implication, or a variation on it. There may be something in this suggestion. For one has the feeling that the basis of our assent to conditionals such as 'If 2 = 3, then 2 + 1 = 3 + 1' is rooted in the existence of a *derivational connection* between 2 = 3 and 2 + 1 = 3 + 1. It seems that the conversational context can in some situations allow us to interpret an if–then statement in an inferential way. Very roughly, the picture might be that if–then statements can have at least *three* logical interpretations. When an if–then statement is used to express a law-like connection, its logical form is explicated by David Lewis' variably strict implication. But an if–then statement can also be used to express a (subjective) conditional expectation of the speaker. In that case, Adams' interpretation in terms of conditional probability is appropriate. Finally, an if–then statement can express an inferential relation. In this case, C.I. Lewis' constantly strict implication, or a variation thereof, supplies the correct logical interpretation. It exceeds the scope of this paper to work out this suggestion in detail.

These options are all left open by our argument. But we maintain on the grounds of the considerations we have brought forward that if conditionals generally have truth-values, then judgements concerning them will be along the lines set out by Lowe. And in that sense, the negative result of this paper seems to offer little room for manœuvre to those who want to defend a variation on C.I. Lewis' proposal for *all* indicative conditionals.[20]

Katholieke Universiteit Leuven

[20] Research for this paper was supported by grant G.0239.02 of the Fund for Scientific Research, Flanders, which is gratefully acknowledged. We are indebted to Igor Douven and to Jonathan Lowe for valuable comments on earlier versions. We are also indebted to an anonymous referee for valuable comments and suggestions for improvement.