A Ranking-Theoretic Approach to Conditionals

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Abstract

Conditionals somehow express conditional beliefs. However, conditional belief is a bi-propositional attitude that is generally not truth-evaluable, in contrast to unconditional belief. Therefore, this article opts for an expressivistic semantics for conditionals, grounds this semantics in the arguably most adequate account of conditional belief, that is, ranking theory, and dismisses probability theory for that purpose, because probabilities cannot represent belief. Various expressive options are then explained in terms of ranking theory, with the intention to set out a general interpretive scheme that is able to account for the most variegated usage of conditionals. The Ramsey test is only the first option. Relevance is another, familiar, but little understood item, which comes in several versions. This article adds a further family of expressive options, which is able to subsume also counterfactuals and causal conditionals, and indicates at the end how this family allows for partial recovery of truth conditions for conditionals.

Keywords: Conditionals; Counterfactuals; Expressivism; Conditional belief; Ranking theory; Ramsey test; Relevance; Truth conditions for conditionals

1. Introduction

It need not be emphasized how important the topic of conditionals is. After it has been underrated and underresearched for more than 2,000 years of intellectual history, attention exploded in the last 50 years, and the topic became almost overresearched. Neither need I emphasize how confusing and confused the topic is. Still, let me start with doing precisely this, hoping to thus make you open for a new approach to the topic. For the confusion is not only due to the intrinsic complexity of the matter, it may as well be caused by the inapt means for coping with the complexity, as I will argue in the article.

The confusion already shows up in the fact that the topic is fragmented among at least four different disciplines with different interests and with often parochial views and

insufficient attention to the neighbors, namely among philosophy, linguistics, psychology, and computer science.

Philosophers are almost obsessively occupied with conditionals since Stalnaker (1968) and Lewis (1973a), both by now well known everywhere, although contemporary discussion starts already with Chisholm (1946) and Goodman (1947). Philosophers are after abstract types of conditionals, their semantic characterization, and their logical behavior; the methodology behind their abstractions is not easy to understand. Still, one might say that most of the basic ideas employed by the other disciplines have been developed in that philosophical discussion. A very deep concern for philosophers, which is little understood elsewhere, is whether conditionals can be characterized by their truth conditions (this is not to say that they are truth functions) or only by something like acceptability or assertibility conditions. A paradigmatic struggle with this concern is found in Stalnaker (1984, chs. 6–8).

To my knowledge, linguists have discovered the topic only with Kratzer (1978). Their interest is to integrate conditionals into compositional semantics, and since truth conditions are their main or only paradigm for compositionality, the pertinence of truth conditions is hardly doubted. However, their further interest, of course, is to seriously come to grips with the linguistic phenomenology. This means to study the semantic interaction of conditionals with moods and tenses, determiners and quantifiers, negation, adverbial constructions, etc., and indeed the interaction of all these linguistic devices with the communicative circumstances and other pragmatic factors—a most demanding business. For a recent survey, see von Fintel (2011).

The interest in semantics is common to philosophy and linguistics. The interest of psychology is different. It is concerned with the actual workings of human thinking and reasoning, and since conditionals play such a salient role, it is clear that the interest extends to conditionals. It seems to have started with Wason (1966), and grew tremendously since (e.g., the impressive bibliography in Evans and Over [2004]), though at first unnoticed by the other disciplines (the first philosophical reference to Wason [1966] seems to be Cohen [1981]). Thus, the psychologists' approach is decidedly empirical. There is, though, a characteristic ambiguity, of which psychologists are well aware (cf., e.g., Evans and Over, 2004, pp. 21ff., pp. 33ff. and Oaksford and Chater, 2007, pp. 30ff.): Empirical work has to refer to some normative standard or rational model, a potential rationalization, which is usually borrowed from philosophy. However, people never fully conform to the standard, there always is at least noise. Hence, psychologists continuously struggle with the issue: Is the deviance from the standard still tolerable? Is there a plausible explanation for it? Or is it so large that the standard itself is defeated? Of course, the answer also depends on the availability of rational models; if there is no better one, deviances tend to get tolerated or explained away in some other way. (I say this, because a major aim of this article will be to provide a new rational model.)

The interests of computer scientists are still quite different, though overlapping. Their need is to implement defeasible or plausible reasoning in the computer, which, as they understood soon, cannot be reduced to deductive reasoning. So, again, they are in the business of model building, but they are restricted neither by empirical adequacy in

general or linguistic adequacy in particular nor by philosophical justification. This is also liberating. Their restriction rather lies in computability. Indeed, the success of Pearl (1988) lies in making probabilistic reasoning, which may surely appear to be the most plausible model, computationally feasible. Due to the liberation we saw many models since the 1970s (for a then up-to-date overview see Pearl, 1988, chs. 9–10), and again interest extended to conditionals, since they are deeply interwoven within defeasible reasoning (non-monotonicity, e.g., being a common characteristic); see, for example, systems Z and Z^+ in Pearl (1990) and Goldszmidt and Pearl (1992) or the logic of counterfactuals within so-called structural models in Pearl (2000, ch. 7).

When four different disciplines with so different interests and so different methodologies address the same topic, confusion is bound to arise. This may also be stimulating. However, disciplinary boundaries are high, and within the boundaries prejudices tend to stabilize. (My references above were also chosen to demonstrate this point; at least some of them, though central, are hardly known outside their field.) This ambivalent situation seems to have changed recently, and the present issue certainly is a further integrating step.

Confusion also shows up in the classification of conditionals we find in the literature. There are indicative and subjunctive conditionals, classified by mood; there are past, present, and future conditionals, classified by tense; there are open conditionals, counterfactuals, and even semi-factual conditionals, apparently classified by the speaker's attitude toward the antecedent and the consequent; there are so-called unconditionals ("there are biscuits on the side board, if you want some"; cf. Merin, 2007) and other strange cases ("if the umbrella is not in the closet, my memory is failing"; cf. Bennett, 2003, p. 340); there are backtracking and non-backtracking conditionals, somehow classified according to what may or may not be assumed along with the antecedent; there are material implications and strict, variably strict, and suppositional conditionals, classified by theoretical descriptions; there are epistemic and causal conditionals, apparently referring to the nature of the relation between antecedent and consequent (as if it would be clear what "causal" means—see section 7 below); Douven and Verbrugge (2010, p. 303) speak of inferential and content conditionals as forming a customary grouping (although they call the latter group "less well delineated"). And so forth.

The topic ramifies further. There are not only conditional assertions but there are also conditional promises, warnings, commands, requests, etc. And there are various conjunctions such as "even if," "although," "because," which are not properly called conditionals, which, however, seem to express the same or very similar relations between the two clauses they connect. The conditional relation is often implicit: "and then" tends to be read as "and therefore"—"post hoc, ergo propter hoc" is not simply a fallacy. And in "you scratch my back, and I scratch yours" (Arthur Merin's example) no conditionality whatsoever is on the surface. Very often, it is condensed in the lexicon: dispositional terms are explained by conditionals, and transitive verbs are often endowed with causal and hence conditional meaning. Indeed, since causation is nearly synonymous with counterfactual conditionals, the conditional idiom extends at least as far as the causal idiom. Conditional content totally penetrates ordinary language.

It is almost impossible to steer clear through all these classifications, classificatory criteria, and ramifications. In this mind-boggling situation it is useful, I think, to step back and reconsider. This is what I shall do in this article. The reconsideration is bound to have modest aims; at each point, my proposals and analyses cannot go deeply into details. Still, by getting back to the basics, I will be able to envisage two strategic changes, which, I believe, can bring orientation and progress to our messy situation.

My program is given in the following: In section 2, I will first briefly argue for the primacy of semantic issues and then plead for expressivism as semantic strategy. The unoriginal suggestion will be that our conditional idiom basically serves to express conditional beliefs. This sounds familiar. However, section 3 will continue to argue that the entire literature on conditionals has relied on, let us say, suboptimal theoretical representations of conditional belief. There is a better one, namely ranking theory, which I shall briefly motivate and introduce. Ranking theory is around since Spohn (1983, 1988); although its utility for the present topic is fairly obvious, it has never been so applied. Huber (2013) and Kroedel and Huber (2013, sect. 3) are exceptions, and maybe Pearl (1990) and Goldszmidt and Pearl (1992), though those papers are rather concerned with developing default logic in ranking-theoretic terms. Applying it here is one of the hopefully beneficial changes.

The other change is developed in sections 4–7. There I shall propose not to look at specific linguistic phenomena and to try to explain them with the new theoretical means. This would be unlikely to succeed, since those explanations will involve many further complexities. Rather, the strategy will be this: If conditionals express conditional belief and if that is best accounted for by ranking theory, what is it that we might express about our conditional beliefs (as represented in ranking theory)? The Ramey test, going back to Ramsey (1929), has been the standard answer, and unfortunately the only one. My idea will be to systematically explore what else we might express about conditional belief, and I will come up with quite a number of illuminating possibilities. So, the second, hopefully beneficial change will consist in considerably extending our explanatory repertoire, even if this goes only halfway to actually explaining specific phenomena. I trust that this change of perspective will turn out to be useful.

2. Why semantics? Why expressivism?

To start from scratch, let me introduce the symbol ▷ for the generic conditional, that is, for any conjunction somehow having conditional content, for anything listed above, even conjunctions like "although" or "because." Considering the generic conditional is justified by our aim to explain a space of possibilities of what conditionals could mean and not to explain the specific meaning of any specific conditional. Therefore, I take a new symbol not yet put to specific use in the relevant literature.

We will thus be considering the sentence schema "$A \triangleright B$." The only restriction I want to assume right away is that I will only look at conditional assertions and not at other illocutionary roles that may be conditionalized as well; assertions are large enough a field.

Starting from scratch is for me starting with semantics, with the *meaning* of conditionals. For analytic philosophers like me this goes without saying. If we do not say, at least provisionally, what conditionals might mean, any further reflection is blocked; the reflection would not even have a subject. Similarly, semantics is a core business of linguistics. Computer scientists, in contrast, are not explicitly concerned with semantics. Still, they have, I think, explicatory claims. When they train their computers in defeasible reasoning, this must be similar to what *we* do; otherwise it would not be justified to call it an implementation of defeasible reasoning.

Psychologists seem least disposed to call their investigations semantic ones. The idea might be that meanings emerge as something like the social or conventional average of individual cognitive states and that inquiry must hence start with the latter. However, even though the social supervenes on the individual level, the explanatory relations certainly go in both directions, in a thoroughly entangled way. The issue of the primacy of the individual over the social (or reversely) need not concern us here.

So let us focus on the semantics of conditionals. Then it is still an open issue which format should be given to semantics. In my second naivety I would say that expressivism is the first choice here. For what is language good for? Primarily for expressing our mental states and attitudes (This emphasizes the speaker's side; the hearer has the complementary task of understanding what is expressed.). In any case, our mental states and attitudes are the immediate causal predecessors of our speech; whatever else it signifies is mediated by them. Thus, the primary meaning of our utterances consists in the states expressed; whatever else they mean is to be inferred from them. This is not to say, of course, that the primary meaning would be the most important or interesting; usually, it is the mediately signified or said that is attended.

This point should favor an expressivist strategy for doing semantics. Expressivism is indeed taken seriously in philosophy, mainly as a metaethical position concerning the meaning of moral sentences but also as a general semantic strategy (cf., e.g., Gibbard, 1990). Still, even in philosophy, truth-conditional semantics is clearly the favored strategy. Why? One point is that semantics must proceed compositionally—this is crucial for linguists—and that we know how to state recursive semantic rules in terms of truth and reference (in possible worlds). In contrast, there are few proposals for a semantic recursion in terms of mental states expressed (cf. e.g., Merin, 1999, 2003, 2006), and they are little acknowledged.

A second point is that the states linguistically expressed are mainly propositional attitudes. Propositions have or, rather, are truth conditions; and belief, the paradigmatic propositional attitude, is truth-evaluable. Thus, truth-conditional semantics carries us most of the way, even if it does not literally apply to imperatives, promises, evaluative idioms, etc., expressing other attitudes. The same point is reflected in speech act theory, which distinguishes illocutionary role and propositional content. Compositional semantics primarily applies to the latter, and assertion expressing belief is a salient illocutionary role, even though there are many others.

A third point, surely the deepest and most difficult, is marked by the so-called linguistic turn, the transition of 18th and 19th to 20th century philosophy, and its insight that

mental states and their contents are identifiable only with reference to external states of affairs. Hence, it seems, we must first study what utterances mediately signify, namely truth conditions, before we can know what they immediately express. Surely, what has been called Gottlob Frege's antipsychologism is the hallmark of that transition. Whatever its present status, it has been extremely healthy and still casts strong doubt on the idea that one could start on the individual level and proceed from there to the social level (cf. also Burge, 1979).

So, without doubt, truth-conditional semantics is fine, as far as it goes. However, it is not good enough for all mental states we might wish to express, not even in the derivative way just mentioned for propositional attitudes besides belief. The exceptions coming to mind first are utterances like "ouch," which expresses pain and has no truth condition (only a sincerity condition, namely actually having pain). If this were the only kind of exception, one might well neglect it. But it is not. I am very sure that conditional belief is a mental state that escapes the truth-conditional strategy as well; conditional beliefs have *no* truth conditions! This is so fundamental that it deserves a label: CB*no*TC. Maybe there are other mental states not amenable to truth conditions; I do not know. However, within our present context CB*no*TC is all-important. Basically, the reason for CB*no*TC lies in the well-known trivialization theorems of Lewis (1976); I will say more about the justification of CB*no*TC in section 4.

For the moment, let us just ponder about the implications of CB*no*TC. First, I have not yet explained what conditional belief really is; this is the task of the next section. It should be clear, however, that it is of fundamental importance for our cognitive life. It governs the dynamics of belief, or, as I should say more cautiously, not the actual dynamics, which is subject to many further influences (such as forgetting), but the rational dynamics and hence all rational learning. The basic rule is that conditional belief turns into unconditional belief upon learning that the condition obtains. Actually this is too crude a learning rule, even from a normative point of view. However, all more sophisticated and more adequate rules for learning and changing beliefs build on the notion of conditional belief. We might also say that all of our non-deductive inferences, all of our inductive strategies depend on our conditional beliefs. One cannot overemphasize their importance for epistemology and cognitive psychology.

A second point is that conditional belief is not obviously definable by unconditional belief. In the case of probabilities this has been forcefully argued by Hájek (2003): What is usually called the definition of conditional probabilities in terms of unconditional ones is only a conditional definition (defining conditional probability only if the condition has positive probability); it thus states only an incomplete relation between conditional and unconditional probability. We will see in the next section that the same holds for belief. Thus, if we find no other ways of reduction, CB*no*TC holds; conditional belief, or conditional probability at least, is not, and is not reducible to, a propositional attitude. It rather is a *bi-propositional attitude*, as it were. Each of the two propositions it relates, the condition and the conditionally believed, is a truth condition; their relation, however, cannot be grasped in truth-conditional terms (more on this in the next sections).

The third point almost goes without saying: Our conditional idiom mainly exists for expressing conditional belief. Again, I should be more cautious. As already remarked, we have various conditional attitudes, and our conditional idiom is designed for expressing all of them. However, as long as we restrict ourselves to assertions, the point holds; assertions express beliefs, and conditional assertions, or assertions of conditionals, express conditional beliefs. As also observed, the latter can also be expressed in many further ways, often quite implicit. Still, the conditional idiom provides the paradigmatic way. So far this seems entirely uncontested in the literature.

The conditional idiom is often qualified to express degrees of belief; we say: If A, then probably, presumably, possibly, it is unlikely that, it is certain that, it may well be that, etc., B. There are many modifiers in natural language indicating strength of belief, at least roughly and vaguely. Introspection does not reveal precise degrees of belief, and hence no more precision may be expected from the expressive means. Because those modifiers are so widespread, it may seem that expressivism must take a probabilistic route, at least as far as assertive speech is concerned. However, we will see in the next section that there are other good accounts of degrees of belief. And then, it is an open question which kinds of degrees of belief are expressed.

In any case, from a semantic point of view those modifiers are additional complexities, and the expression of degrees of belief seems to be committed to such modifiers. Hence, the basic, unmodified conditional idiom expresses conditional belief *simpliciter* and not any degrees thereof. And the basic case is the one to be addressed first.

The upshot of this section is given in the following: If CB*no*TC is right and therefore expressivism the semantic strategy to be employed, then any investigation of conditionals must start with studying conditional beliefs and the expressive relation between conditionals and conditional beliefs. This is what I shall do in the rest of the article.

The upshot is not new, of course. It is embodied in the Ramsey test, which derives from Ramsey (1929, pp. 142ff.) and which directly correlates conditionals with conditional or suppositional beliefs. We will see that there are many more expressive options. Moreover, the Ramsey test is rather only a guiding idea that has found various explications in the literature. So disagreement will start when we obtain to the details.

3. Conditional belief

How should we account for conditional belief? Let us at least introduce symbols: $\mathbf{B}(A)$ represents unconditional belief in A, and $\mathbf{B}(B \mid A)$ represents conditional belief in B given or conditional on A. The subject and the time of belief may be left implicit. Here, A and B stand for propositions, not for sentences. A proposition is a set of possibilities and thus represents a truth condition of a sentence, that is, the set of possibilities in which the sentence is true.

To be precise, let W be the set of all possibilities in a given case (you may, but need not think of possibilities as full possible worlds), and let $\mathcal{A}$ be an algebra of subsets of W (which is closed under negation, conjunction, and disjunction). $\mathcal{A}$ is the set of propositions

at hand, and A and B are taken from $\mathcal{A}$. $\bar{A}$ is the negation of A, $A \cap B$ the conjunction of A and B, $A \cup B$ their disjunction, and $A \rightarrow B = \bar{A} \cup B$ the material implication (in set-theoretical instead of sentential terms). For our purposes, it does not really matter whether we speak of propositions or sentences, and sometimes I may sloppily interchange them.

How are we to account for conditional belief? A small minority (e.g., Jackson, 1987; Johnson-Laird & Byrne, 1991) still defends the view that the conditional $A \rhd B$ may basically be interpreted as the material implication $A \rightarrow B$, thus expressing $\mathbf{B}(A \rightarrow B)$, the belief in the material implication. This view was more popular in the past, I think, simply in default of theoretical alternatives; for its half-truth see below. However, nobody has proposed to identify the conditional belief $\mathbf{B}(B \mid A)$ with the unconditional belief $\mathbf{B}(A \rightarrow B)$. This would be crazy; for, if we take A to be false, we take $A \rightarrow B$ as well as $A \rightarrow \bar{B}$ to be true (this is one of the paradoxes of material implication), and then, according to this proposal, we would believe B as well as $\bar{B}$ conditional on A. However, even conditional belief must rationally be consistent (under all entertainable conditions). Hence, the proposal is inadequate.

Certainly, the most popular view today is to treat conditionality in terms of conditional (subjective) probabilities; at least philosophers (e.g., Adams, 1965, 1975; Edgington, 1995, 2003, 2008) and psychologists (e.g., Oaksford & Chater, 2007) are very fond of this idea. (Linguists are not at all; they would not know how to do semantics in probabilistic terms.) I agree halfway; conditional probabilities indeed provide about the best model of conditionality we have. The problem, however, is that belief is not probability, and conditional belief is not conditional probability.

The most plausible connection between belief and degree of belief is that belief is sufficient degree of belief; this is called the Lockean thesis (by Foley, 1992), and always interpreted in terms of probabilities. However, thus interpreted, the Lockean thesis is simply untenable. The basic point is given in the following: It is a fundamental law of rational belief that, if you believe A and believe B, you also believe $A \cap B$; believing is the same as taking to be true. However, if your probability of A is high (above the relevant threshold) and that of B is so, too, that of $A \cap B$ need not be. Thus, this fundamental law of rational belief refutes the probabilistic Lockean thesis. This point is highlighted by the well-known lottery paradox (cf. Kyburg, 1961, p. 197). The issue has provoked a vigorous discussion with quite a few epicycles, which I cannot go through here. My conclusion is that there is no good way to save the probabilistic Lockean thesis. The unavoidable consequence is a certain epistemological schizophrenia: We have beliefs and we have probabilities, and neither is reducible to the other. A desperate move might be to skip speaking of belief at all (this is radical probabilism, as defended by Jeffrey [1965] and in his later writings). However, I am sure that this is the worse strategy. (For all this see my extensive discussion in Spohn [2012, sect. 3.3, and ch. 10].)

The point extends, of course, to conditional belief and probability. Everyone accepts the following logical law for indicative (and subjunctive) conditionals: If $A \rhd B$ and $A \rhd C$, then $A \rhd B \cap C$. As far as I know, this law has not been put to empirical test. As a normative law, it seems beyond doubt. It fits nicely to the Ramsey test that takes

conditionals to express conditional beliefs, because everyone accepts the following law of rational conditional belief: if $\mathbf{B}(B \mid A)$ and $\mathbf{B}(C \mid A)$, then $\mathbf{B}(B \cap C \mid A)$. However, we get none of this, if we identify conditional belief with high conditional probability; the lottery paradox raises its ugly head again. Hence, it seems inadequate to treat conditionals and conditional belief in probabilistic terms.

This seems to contradict Adams (1965, 1975), who has developed the standard logic of conditionals (including the above law) in probabilistic terms in a most attractive way. There is not really a contradiction. Adams starts with what has been called *Adams' thesis*, or simply *the equation* (following Edgington, 1995, p. 271): that "the probability of an indicative conditional of the form 'if A is the case then B is' is a conditional probability" (Adams, 1975, p. 3), that is, $P(A \triangleright B) = P(B \mid A)$, provided A and B do not contain a conditional in turn, that is, belong to factual language, as Adams says. This is a probabilistic version of the Ramsey test and perceived as such everywhere. However, in view of the criticism of Lewis et al. (1976) (see here section 4) it must be doubted that $P(A \triangleright B)$ can be called the probability of the conditional; it is better, though quite obscurely called its assertibility or acceptability value.

In itself, Adams' thesis provides neither a semantics nor a logic for $A \triangleright B$. However, Adams ingeniously turns this into a criterion of validity of logical inferences with conditionals: "if an inference is truth-conditionally sound, then the uncertainty of its conclusion cannot exceed the sum of the uncertainties of its premises" (Adams, 1975, p. 3). The formal version is this: An inference is sound if and only if for each $\varepsilon > 0$ there is a $\delta > 0$ such that for all probability measures, if all of its premises have probability $\geq 1-\delta$, its conclusion has probability $\geq 1-\varepsilon$ (cf. Adams, 1975, p. 57). Thereby, Adams is able to account for the logical behavior of indicative conditionals in the widely accepted standard form.

All this is very nice, and in a way I have no quarrel with it at all. Adams succeeds because he ingeniously exploits the behavior of conditional probabilities for his logical purposes with his ε, δ-limit constructions. Thereby, he does not equate conditional belief with conditional probability 1; this would have been intuitively inadequate and formally defective (because probabilistic reasoning does not work properly with extreme probabilities). However, he can thus approximate the behavior of conditional belief.

Still, I have three reservations: First, my proposal below will capture conditional belief directly; there is no need at all for probabilistic detours and those quite involved ε, δ-limit constructions. We can thus avoid Adams' false pretense that we could approach belief by approaching probability 1. Moreover, Adams, though properly accounting for the logic of conditionals, does not really provide a semantics for conditionals, not even an expressivistic one. Strictly speaking, he has an expressivistic account only of conditionals qualified by epistemic modifiers and not of conditionals *simpliciter*. Finally, subjective probabilities cannot be true or false. So Adams cannot point any way for conditionals to be true or false. Intuitively, however, at least some conditionals can be true or false; it would be embarrassing to deny this. If we approach conditionals in terms of belief instead of probabilities, we may have access to truth values, because beliefs can be true or false in an unproblematic way. I shall say more about this issue in sections 6–8.

So let us put probability to one side. How else could we grasp conditional belief? Curiously, so-called belief revision theory, which has developed into a huge research field, originates directly from the Ramsey test. Gärdenfors (1979, 1981) was motivated by this test to inquire the rational behavior of belief revision, of what to believe after supposing or accepting a possibly belief-contravening proposition. This has developed into so-called AGM belief revision theory (according to Alchourrón, Gärdenfors, & Makinson, 1985). This theory is canonized in Gärdenfors (1988); it has, however, found many hotly debated variants (cf., e.g., Rott, 2001, sect. 4.2-3).

The importance of this theoretical field cannot be overemphasized. It was about the first genuine emancipation from the probabilistic paradigm. However, very early, in Spohn (1983, sect. 5.2, and 1988, sect. 3), I raised a decisive criticism: Belief revision theory does not provide a complete dynamics of belief; it can account for the first change, but it has no theoretical means for accounting for further changes. This is called the problem of iterated belief change. It has been thoroughly attended. Still, up to now, all solutions within the confines of belief revision theory have been quite incomplete (see my detailed discussions in Spohn, 2012, sect. 5.6 and ch.8).

I have proposed a solution that fully solves the problem of iterated belief change in Spohn (1983, sect. 5.3, and 1988, sect. 4) by what is now called ranking theory (this is a suitable term, but it refers to different theories in economics and elsewhere); the theory is fully developed, explained, and defended in Spohn (2012) and partially in many earlier papers. Let me introduce the formal definition and then explain it; it is crucial for the rest of the article.

Definition: κ is a *negative ranking function* for $\mathcal{A}$, the algebra of propositions over W, if and only if κ is a function from $\mathcal{A}$ into $N \cup \{\infty\}$, the set of natural numbers plus infinity, such that for all $A, B, \in \mathcal{A}$:

$$\kappa(W) = 0 \text{ and } \kappa(\varnothing) = \infty \tag{1}$$

$$\kappa(A \cup B) = \min \kappa(A), \kappa(B). \tag{2}$$

$\kappa(A)$ is called the (*negative*) *rank* of A. If $\kappa(A) < \infty$, then the *conditional rank* of B given A is defined as

$$\kappa(B \mid A) = \kappa(A \cap B) - \kappa(A). \tag{3}$$

Negative ranks represent degrees of *disbelief* (this is why I call them negative). That is, $\kappa(A) = 0$ says that A is not disbelieved, and $\kappa(A) = n > 0$ says that A is disbelieved (to degree n). According to (1) and (2) we have min $\{\kappa(A), \kappa(\bar{A})\} = \kappa(W) = 0$. That is, you cannot take both, A and $\bar{A}$, to be false. But we may have $\kappa(A) = \kappa(\bar{A}) = 0$, in which case κ has no opinion about A. *Belief* in A, $\mathbf{B}(A)$, is the same as disbelief in $\bar{A}$ and thus represented by $\kappa(\bar{A}) > 0$. Similarly for conditional ranks. $\kappa(B \mid A) = 0$ says that B may be true given A according to κ. $\kappa(B \mid A) > 0$ expresses disbelief in B given A. $\kappa(\bar{B} \mid A)$ represents *belief* in B given A, that is, $\mathbf{B}(B \mid A)$.

The crucial point is that beliefs may be weaker or firmer and they are still beliefs. This is our everyday notion, and it is respected by ranking theory. Ranks measure those degrees of beliefs (indeed, there is a rigorous measurement theory for ranks, just as for probabilities, the difference being that ranks are measured on a ratio scale and probabilities on an absolute scale—cf. Hild and Spohn [2008] and Spohn [2012, ch. 8]). Thus, we are dealing with two different kinds of degrees of belief, ranks and probabilities, and only one of them also represents belief. On the other hand, there is also a striking similarity between the ranking axioms (1)–(3) and the axioms of probability (including the definition of conditional probability); by taking the logarithm of probabilities relative to a small (or infinitesimal) base the latter roughly (or exactly) translate into the former. This generates many mathematical similarities despite the strict interpretational difference. (For a rigorous translation see Spohn [2012, sect. 10.2].)

One might also say that Adams probabilistically approximates the behavior of conditional ranks with his ε, δ-limit constructions. However, it is so much more straightforward to simply replace probabilities by ranks. This is, in a nutshell, what I shall propose.

To make ranking theory more vivid, let me cast good ol' Tweety into ranking terms: Tweety has, or fails to have, each of three properties: being a bird (B), being a penguin (P), and being able to fly (F). This makes for eight possibilities. Suppose you have no idea who or what Tweety is (perhaps it is a book). Your negative ranking function might thus have the qualitatively plausible shape shown in Table 1.

Then, for instance, $\kappa(\bar{B} \cap P) = \min\{11, 8\} = 8$; that is, you very firmly take it to be false that Tweety is a penguin, but not a bird (though you do not take it to be impossible); in other words, you firmly believe that Tweety is a bird, if it is a penguin (taken as a material implication). The strongest proposition you believe is the disjunction of the 0 entries, that is, that Tweety is *either* not a penguin and not a bird ($\bar{B} \cap \bar{P}$) *or* a flying bird, but not a penguin ($F \cap B \cap \bar{P}$); all other possibilities are disbelieved in κ.

So you also believe everything logically entailed by that strongest proposition, for instance $B \rightarrow (\bar{P} \cap F)$ ($\rightarrow$ = material implication), and both $P \rightarrow F$ and $P \rightarrow \bar{F}$, but only because you believe $\bar{P}$, that Tweety is not a penguin, anyway.

If we look at the conditional beliefs contained in the example, we discover further interesting features. Not all material implications believed translate into conditional beliefs, but some do (see below for the formal relation). For instance, you believe that Tweety is not a penguin and can fly given it is a bird (since $\kappa(P \cup \bar{F} \mid B) = \kappa(B \cap (P \cup \bar{F})) - \kappa(B) = \min\{4, 2, 1\} - \min\{0, 4, 2, 1\} = 1 > 0$). And you believe that Tweety cannot fly given it is a penguin (since $\kappa(F \mid P) = 3$), but not that it can fly given it is

Table 1
Tweety

κ	$B \cap \bar{P}$	$B \cap P$	$\bar{B} \cap \bar{P}$	$\bar{B} \cap P$
F	0	4	0	11
$\bar{F}$	2	1	0	8

a penguin (since $\kappa(\bar{F} \mid P) = 0$). You also believe that it is not a penguin given it is a bird (since $\kappa(P \mid B) = 1$), and reversely that it is a bird given it is a penguin (since $\kappa(\bar{B} \mid P) = 7$). And so forth.

I have only defined conditional ranks and not specified any ranking-theoretic learning rules. However, it is at least plausible that those rules may be stated in analogy to probabilistic rules and that those rules may be iteratively applied (thus solving the problem of iterated belief change). Without doubt, the Tweety example would then find an illuminating continuation. (For all details see Spohn [2012, ch.5].)

You may still complain about those negative ranks expressing disbelief. Even the Tweety example was not perfectly smooth reading, precisely because of these negative ways. This is completely outweighed, however, by the tremendous theoretical advantages generated by the comprehensive analogy to probability. And this analogy is displayed only in terms of disbelief and not in terms of belief.

It is, however, no problem at all to directly represent belief. If κ is a negative ranking function for $\mathcal{A}$, we may define the *positive ranking function β* for $\mathcal{A}$ by:

$$\beta(A) = \kappa(\bar{A}). \tag{4}$$

β thus expresses degrees of *belief*; $\beta(A) > 0$ says that A is believed and $\beta(A) = 0$ says that A is not believed (this is not to say that A would be disbelieved). We may also directly axiomatize positive functions by translating (1) and (2) by (4), that is,

$$\beta(W) = \infty \text{ and } \beta(\varnothing) = 0, \tag{5}$$

$$\beta(A \cap B) = \min \{\beta(A), \beta(B)\}. \tag{6}$$

Definition (6) says that your degree of belief in a conjunction equals your weakest degree of belief in the conjuncts—and thus entails that you believe a conjunction if and only if you believe both conjuncts. To repeat, this basic feature of rational belief cannot be made intelligible on purely probabilistic grounds. The definition (3) of conditional negative ranks translates into

$$\beta(B \mid A) = \beta(A \rightarrow B) - \beta(\bar{A}). \tag{7}$$

At first an unlikely translation, but its content is quite intuitive, saying that your degree of belief in a material implication is your corresponding conditional degree of belief plus your degree of belief in the vacuous truth of the implication, that is, in the falsity of its antecedent. I will unfold the importance of (7) below. However, with (7) it is particularly clear that positive ranks have no analogy with probabilities. Hence, my determined preference for negative ranks.

We may even integrate positive and negative ranks into one notion, which I call a *two-sided ranking function* τ defined by the following:

$$\tau(A) = \beta(A) - \kappa(A) = \kappa(\bar{A}) - \kappa(A). \tag{8}$$

Conditional two-sided ranks are defined analogously:

$$\tau(B \mid A) = \beta(B \mid A) - \kappa(B \mid A) = \kappa(\bar{B} \mid A) - \kappa(B \mid A). \tag{9}$$

Two-sided ranks are intuitively most intelligible, because they express belief and disbelief at once: A is believed or disbelieved or neither iff, respectively, $\tau(A) > 0$, < 0, or $= 0$. Similarly for conditional two-sided ranks. This is why I shall often refer to two-sided ranks below. However, the formal behavior of two-sided ranks is clumsy; it is only accessible via definition (8) and the formal behavior of the component concepts.

The long and the short of all this: If we want to theoretically capture (rational) conditional belief, we best do it by (3) and (7). Positive and negative conditional ranks are, respectively, conditional degrees of belief and disbelief, and if these degrees are non-zero, they even represent conditional belief and disbelief. I have not fully defended my claim that this is the optimal representation; the offers on the market are too variegated. At least, I have briefly explained my preference over some of the main alternatives. In a way, my entire book (Spohn, 2012) is the richest defense I can give. So we best continue our expressivistic investigations of conditionals in terms of this representation.

It may seem that ranking theory is just one of many attempts to somehow capture non-probabilistic degrees of uncertainty. There is no point of rehearsing here the manifold of plausibility measures, or whatever the pertinent labels are (for a systematic overview see Halpern, 2003). The field is confusing, no theory seems to stand out clearly, and other fields, in particular psychology, receive no good guidance what to make of all this. Thus, the inclination to stick to the established degrees of uncertainty, that is, probabilities, is strong. This is, for instance, the conclusion of Oaksford and Chater (2007, pp. 73f).

However, this would be a misperception of ranking theory. Ranking theory is the only theory within this manifold that explicitly sets out to represent belief and the dynamics of belief. And its normative foundations are utterly strong. They do not consist in shaky intuitions about the behavior of degrees of uncertainty such as Theophrastus' rule (= the weakest link principle, which holds in ranking theory only under severe constraints; cf. Spohn, 2012, theorem 11.30). Rather, in view of the definition (3) of conditional ranks the main axiom (2) is equivalent to

$$\text{either } \kappa(B \mid A) = 0 \text{ or } \kappa(\bar{B} \mid A) = 0 \text{ or both.} \tag{10}$$

Definition (10) simply requires your conditional beliefs to be consistent as well; given A, you cannot disbelieve, and thus believe, both B and $\bar{B}$. Hence, the entire theory follows from (10) and (3) (and (1), which just specifies the range of ranks). These normative

foundations are not easily dismissed. And, of course, they are also empirically relevant; such basic norms like (conditional) consistency surely guide the actual behavior of people (though not always perfectly). (For more extensive comparisons see Spohn [2012, ch.11].)

4. The Ramsey test

In the sequel, I shall explain various expressive options for the generic conditional ▷, more than 20, if one counts in a fine-grained way. So there will be much more to look at than the Ramsey test. On the basis of ranking theory, the logic of all these options and their interpretations could be rigorously developed. This is not my aim here. Rather, I want to make clear that all these options are actually used; they have important and widespread exemplifications in ordinary discourse. This will sufficiently occupy me. And my suggestion will be twofold: that linguists will find a rich analytical tool for systematically capturing the variegated usage of conditionals in ordinary language, and that psychologists will find a rich spectrum of options for alternative interpretations of their experiments and data.

Surely, though, we should start with the *Ramsey test*. According to it, the conditional $A \vartriangleright B$ expresses

$$\tau(B \mid A) > 0, \tag{I.1}$$

that is, $\mathbf{B}(B \mid A)$, the conditional belief in B given A, or under the supposition of A. Some speak here of the assertibility or of the acceptability conditions of $A \vartriangleright B$, and some even see subtle differences between them (cf. Douven and Verbrugge [2010] for a more careful discussion). This manner of speaking is primarily intended to mark an opposition to truth conditions. I do not find it very clear or helpful, though, and prefer to continue speaking of what conditionals express.

I need not rehearse the impressive plausibility of, and evidence for, the thesis that indicative conditionals are often characterized by the Ramsey test, that is, express (I.1). I mentioned Adams's thesis that $P(A \vartriangleright B) = P(B \mid A)$ (where $P(A \vartriangleright B)$ may be interpreted as the degree of acceptability of $A \vartriangleright B$), which is just a probabilistic version of the Ramsey test. Its wide acceptance is part of that evidence. I should also mention that the logic of ▷, which is easily derived from (I.1) without Adams' limit constructions, is just the standard one, including the non-monotonicity of ▷.

At this point I can explain the central thesis CB*no*TC in a bit more detail: that conditional belief as stated in (I.1) has no truth conditions. There is also what has been called *Stalnaker's thesis* (cf. Stalnaker, 1970, sect. 3), which is subtly different from Adams' thesis. Both claim that $P(A \vartriangleright B) = P(B \mid A)$. The difference is that Adams restricts A and B to belong to factual language, whereas Stalnaker does not. The difference is indeed huge. Stalnaker thereby assumes $A \vartriangleright B$ to be a proposition of its own, having or being a truth condition in turn, the unconditional probability of which is explained by this thesis—with the effect that the conditional can be nested. No such assumption is made by Adams,

whence his term $P(A \triangleright B)$ should not be interpreted as the probability of a proposition (and perhaps not as a probability at all).

So Stalnaker (1970) assumes CB*no*TC to be false. However, his thesis has been badly refuted by Lewis et al. (1976), called the "bombshell" by Edgington (1995, p. 271): the proof that only most trivial probability measures can satisfy Stalnaker's thesis. Even more pertinent is the impossibility result of Gärdenfors (1986), which transfers the "bombshell" to belief revision theory and thus to the original qualitative or non-probabilistic version of the Ramsey test. Omitting the detour via conditionals and thus not arguing with embedded conditionals, I directly show in Spohn (2012, sect. 15.3) that conditional belief is not objectivizable, that is, cannot generally be assigned truth conditions. The latter therefore seems to be the more elegant result; however, it rests on my objectivization theory in Spohn (2012, ch. 15). In any case, we should accept CB*no*TC and hence the fact that an expressivistic semantics of conditionals cannot be reduced to truth-conditional semantics.

How does this agree, though, with the *equivalence thesis*, which says that at least the indicative conditional $\triangleright$ is simply the truth-functional material implication $\rightarrow$, perhaps amended by some suitable pragmatic background (a position prominently suggested by Grice [1975] and further defended by Lewis [1976, pp. 305ff.], Jackson [1987], and Johnson-Laird and Byrne [1991])? It does not agree at all. However, the half-truth of this position is well explained by (I.1). With the definitions (8) and (9), (7) immediately entails:

$$\text{if } \tau(A) \geq 0, \text{ then } \tau(B \mid A) > 0 \text{ if and only if } \tau(A \rightarrow B) > 0. \qquad (11)$$

That is, if A is not taken to be false, B is believed conditional on A if and only if the corresponding material implication $A \rightarrow B$ is believed, that is, if not $\mathbf{B}(\bar{A})$, then $\mathbf{B}(B \mid A)$ if and only if $\mathbf{B}(A \rightarrow B)$. In still other terms, if not $\mathbf{B}(\bar{A})$, $A \triangleright B$ may be taken to express either belief, since they amount to the same thing.

So the equivalence thesis that states unconditional identity is false. However, one might say that it is true under the assumption that the antecedent is not taken to be false. This assumption thus is an adequate epistemic characterization of the required pragmatic background. One might also say that the equivalence thesis holds for so-called open conditionals (see (II) below), which are indicative conditionals additionally characterized by the speaker being indeterminate about the antecedent (i.e., $\tau(A) = 0$). According to (11), being open in this sense is not required; $\tau(A) \geq 0$ is enough of an assumption. But note that open conditionals are thereby only epistemically and not in any way linguistically identified.

The equivalence thesis derives its plausibility from the innocent principle often called the direct argument: "A or B" entails "if not A, then B." This appears most convincing, and indeed has many correct instances. Jackson (1987, sect. 1.1) calls this the passage principle and proves the equivalence thesis from it and two further, even more innocent principles. However, (11) explains what is wrong with the direct argument; the entailment holds only if the disjunctive premise "A or B" is open as well in the sense that it is not assumed because A itself is already taken to be true. That is, the direct argument itself holds only conditionally, and no unconditional equivalence may be derived from it. This

well agrees with the account of Stalnaker (1975, sect. IV) of the direct argument. And Evans and Over (2004, p. 114) point to the same fact, when they say that it makes a difference whether one has a constructive or a non-constructive justification, as they call it, for the disjunctive belief "*A* or *B*." If having a non-constructive justification for "*A* or *B*" only means believing the disjunction without believing any of the disjuncts, then their explanation comes to the same.

Note, by the way, that Jackson (1987, pp. 22–32) takes great pains to make his equivalence thesis compatible with Adams' thesis—which it does not seem to be, since we can only prove that $P(A \rightarrow B) \geq P(B \mid A)$. According to (11) no such additional efforts are needed in our ranking-theoretic setting.

If (I.1) is an expressive option, then we might use the generic conditional $A \rhd B$ also for expressing:

$$\tau(B \mid A) = 0, \text{ or} \tag{I.2}$$

$$\tau(B \mid A) < 0, \tag{I.3}$$

or combinations thereof. For instance, indicative might-conditionals usually express (I.2 or I.3), that is, $\tau(B \mid A) \geq 0$; "if it rains, I may come late" expresses that I do not believe to be in time given it rains (of course, this non-belief does not amount to the belief that I will not be in time, if it rains). Note also that (I.3) is the same as (I.1) with *B* replaced by $\bar{B}$.

The Ramsey test deserves its prominence, but it does not deserve exclusive interest. In asserting $A \rhd B$ there are many more attitudes toward *A* and *B* and their epistemic relation we might wish to express. Before turning to more substantial options, let us not forget that in uttering $A \rhd B$ we might also express our attitudes toward *A* by itself and toward *B* by itself, that is, whether

$$\tau(A) > 0, \tag{II.1}$$

$$\tau(A) = 0, \text{ or} \tag{II.2}$$

$$\tau(A) < 0, \tag{II.3}$$

and whether

$$\tau(B) > 0, \tag{III.1}$$

$$\tau(B) = 0, \text{ or} \tag{III.2}$$

$$\tau(B) < 0, \tag{III.3}$$

that is, whether we take the antecedent and the consequent of the conditional to be true or to be false. We do not need the conditional idiom to express (II) or (III) by itself, but using it is mostly accompanied by (II) and (III). I already mentioned open conditionals that are defined to express (II.2) (and usually express [III.2] as well).

If counterfactuals deserve their name, we thereby express that we take their antecedent and their consequent to be false, that is, as counterfactual. However, no rule without exception. By saying "if he had taken arsenic, he would have shown exactly those symptoms which he does in fact show" (Anderson, 1951), I express my belief that he did take arsenic—and indeed, I make an inference from the truth of the consequent to the truth of the antecedent (see option [IV] below for the possible correctness of such an inference). So I will continue speaking of conditionals *usually* expressing this and that. It is extremely difficult for linguists to state stricter rules, and I will not engage in this business, though I hope to facilitate it by extending the expressive options.

Other examples for (II) and (III) are "even if" and "because," if we subsume them under the generic conditional. "*B* because *A*" expresses, among other things, (II.1) and (III.1), that is, belief in *A* and in *B*, while "*B* would have been the case, even if *A* had been the case" usually expresses, among other things, (II.3) and (III.1) and is thus also called a semi-factual. With a little ingenuity one presumably finds instantiations for all nine combinations of (II.1–3) and (III.1–3).

As mentioned at the end of section 2, we cannot only express belief in *A*, etc., but also degrees of belief in *A*, with the help of various modifiers. I have explained in section 3 that these degrees need not be identified with probabilities; they might also be construed as ranks. However, sensitivity to degrees of belief must not blind us to the fact that the basic phenomenon to be expressed is belief itself. Here, I shall not further ponder about the expression of strength of belief, because it is not specific to the conditional idiom; the relevant modifiers are equally common in unconditional assertive speech.

5. Relevance

It should have been clear that the attitudes toward the antecedent and the consequent, which I have just discussed, are only a minor part of what is expressed by a conditional. The major part concerns their epistemic relation. The Ramsey test is one such relation, but only a first glimpse. Another such relation, a most important one indeed, is *epistemic relevance*. Five decades ago or so, relevance was a residue left to the pragmatic wastebasket, but only because there was no way to capture relevance with the means of extensional logic. A nice example for this ignorance is found in many old logic texts, which say that "but" is basically, that is, truth-functionally equivalent to "and," whereby the entire wit of "but" is missed. (For a monograph on how to do better, see Merin [1996].) A graver example is the celebrated Hempel–Oppenheim theory of deductive-nomological explanation, which foundered precisely at its inability to incorporate relevance considerations.

Philosophers tried various ideas to understand relevance (one idea being relevance logic; cf. Anderson & Belnap, 1975). Sperber and Wilson (1986) is very well received in linguistic pragmatics and also in psychology. However, as illuminating their observations on the role of relevance in human communication are, as empty is their general characterization of what relevance basically is: "An assumption is relevant if and only if it has some contextual effect in that context" (p. 122). Evans and Over (1996) are on a more

substantial track when they equate relevance with some form of epistemic utility, though I find the allusion to epistemic decision theory problematic. In my view, the epistemically basic sense of relevance is captured in (subjective) probability theory by its notion of (in-) dependence: A is relevant to B if and only if B probabilistically depends on A, that is, if $P(B \mid A) \neq P(B \mid \bar{A})$, that is, if A makes a difference to the epistemic assessment of B. Clearly, we can also distinguish positive and negative relevance. This is indeed the basic notion of inductive logic and confirmation theory (cf., e.g., Carnap, 1971).

Here, this probabilistic notion is not useful, since I have abandoned probabilistic accounts of conditionals. However, there is a ranking-theoretic analog. A is *positively relevant*, *irrelevant*, or *negatively relevant* to B, if and only if, respectively, A raises, does not change, or lowers the degree of belief, that is, the two-sided rank of B, that is, if respectively,

$$\tau(B \mid A) > \tau(B \mid \bar{A}), \tag{IV.1}$$

$$\tau(B \mid A) = \tau(B \mid \bar{A}), \tag{IV.2}$$

$$\tau(B \mid A) < \tau(B \mid \bar{A}). \tag{IV.3}$$

(For my reasons for stating, for example, [IV.1] as above and not as $\tau(B \mid A) > \tau(B)$, see Spohn [2012, pp. 106f.].)

Note that an adequate representation of relevance as in (IV) requires the full resources of ranking theory, which were not yet required for the Ramsey test (I). One cannot adequately establish the kind of comparison required for (IV) on the basis of purely ordinal conceptions such as the entrenchment orderings of belief revision theory (cf. Gärdenfors, 1988, ch. 4) or the similarity spheres of Lewis (1973a). This is a crucial point in my view why ranking theory is superior to those alternative theories.

So my suggestion is that the generic conditional ▷ may be used to express some kind of relevance (IV.1–3)—and indeed is mostly so used. I think the Ramsey test without relevance is not very common. The indicative conditional "if" usually expresses positive relevance. When I say "if it rains, the plants will die," I thereby express that I do not believe that the plants will die, anyway. "If Oswald did not shoot Kennedy, someone else did" clearly expresses positive relevance: given Oswald did shoot Kennedy, I do not believe, or believe less firmly, that someone else did as well. (More on this famous example below.) "Because" also expresses positive relevance; it differs from "if" only with respect to (II) and (III). "The plants died, because it rained" does express the same as before (after one learned that it rained and the plants died). (More on "because" below.) There are many more ways to express positive relevance (IV.1).

"The plants will die whether or not it rains" clearly expresses irrelevance (IV.2). And negative relevance (IV.3) may have even more expressive means than positive relevance. "Despite" basically indicates negative relevance. So does "but." It is generally deviant to say "Fa, but Fb," for instance, "Ann sings, but Bob sings" (whereas "Fa and Fb," "Ann sings and Bob sings" is perfectly okay). A good explanation lies in Carnap's principle of positive instantial relevance (cf. Carnap, 1971, sect. 13) that, in the absence of further

background information, one instance of a feature makes it more likely that we next encounter a further instance of that feature. Therefore, the "but" expressing negative relevance is odd. (For this observation see Merin [1996, 1999].) "The plants died, although it rained" expresses that, given it rained, it came as a surprise that the plants died; one would rather have expected them to die without rain. From the expressivist point of view, relevance is a central epistemic aspect to be expressed and not merely some pragmatic add-on.

Even in psychological experiments, positive relevance often seems to be an implicit assumption. For instance, in their experiments 1 and 2, Over, Hadjichristidis, Evans, Handley, and Sloman (2007) asked their participants to estimate the probability of the four truth value distributions over the antecedent and the consequent, for a large variety of conditionals, which were classified according to whether the probability of the antecedent and, respectively, of the consequent was rated high or low. In all the cases thus distinguished, the antecedent was, in the average, considered positively relevant to the consequent; that is, the probability of the consequent given the antecedent exceeded that given the negation of the antecedent by .23–.32. This is what makes the conditionals meaningful. I wonder what the results of the experiments would have been, if the participants had been offered non-sensical conditionals like "if the Queen resigns, the Iran will stop its nuclear program," where the antecedent is obviously irrelevant to the consequent.

Note that the expressive options (II), (III), and (IV) are logically independent; unconditional degrees of belief in A and B are compatible with any direction of relevance between A and B. It would be interesting to develop the logic of relevance conditionals in the sense of (IV.1). Let me only remark that the properties of positive relevance are not at all straightforward and not completely known (see Spohn, 2012, though theorem 6.7 there states most of those properties). I should point out, though—note that this "though" indicates that I am going to say something contrary to your expectations raised by "let me only remark"—two basic features.

First, ranking theoretic positive relevance is *symmetric*:

$$\text{if } \tau(B \mid A) > \tau(B \mid \bar{A}), \text{ then } \tau(A \mid B) > \tau(A \mid \bar{B}); \qquad (12)$$

that is, if A is positively relevant to B, B is positively relevant to A.

Second, it holds for the negations as well; that is,

$$\text{if } A \text{ is positively relevant to } B, \bar{A} \text{ is positively relevant to } \bar{B}. \qquad (13)$$

(13) combined with (12) yields contraposition (with respect to relevance):

$$\text{if } A \text{ is positively relevant to } B, \bar{B} \text{ is positively relevant to } \bar{A}. \qquad (14)$$

The same applies to negative relevance. It is well known that the very same claims hold for probabilistic relevance or dependence.

This casts some light on a phenomenon that puzzled psychologists very much. They studied four patterns of inferences:

$$\textit{Modus Ponens} \ (\text{MP}): \quad \text{If } A, \text{ then } B. \ A. \text{ Therefore } B.$$

$$\textit{Modus Tollens} \ (\text{MT}): \quad \text{If } A, \text{ then } B. \ \bar{B}. \text{ Therefore } \bar{A}.$$

$$\textit{Affirming the Consequent} \ (\text{AC}): \quad \text{If } A, \text{ then } B. \ B. \text{ Therefore } A.$$

$$\textit{Denial of the Antecedent} \ (\text{DA}): \quad \text{If } A, \text{ then } B. \ \bar{A}. \text{ Therefore } \bar{B}.$$

If "if, then" is interpreted as material implication, then clearly (MP) and (MT) are valid, while (AC) and (DA) are fallacious. Therefore, psychologists were irritated to find strong inclinations among subjects to endorse all four inferences (though [MP] came out most frequent and [DA] least frequent—for some of the many data see, for example, Evans & Over, 2004, pp. 45ff.).

An old explanation was the "chameleon" theory of Braine (1978) that "if" tends to be read as "if and only if" (pronounced "iff"). But whence this tendency? It is not mere caprice. Therefore, the deeper Bayesian explanation seems favored nowadays (cf. Oaksford & Chater, 2007, pp. 118ff.), which interprets inference and "if" in terms of positive relevance and observes that one positive relevance statement entails the other three (according to the probabilistic versions of [12]–[14]). It makes room, moreover, for explaining the varying tendencies to endorse those inference patterns. For, although one positive relevance statement entails the other three, they need not be equally strong; certainly, the stronger the positive relevance, the stronger the tendency to accept the corresponding inference.

However, here we wanted to do without the probabilistic representation. Therefore, I would like to point out that precisely the same explanation is available in ranking-theoretic terms, as (12)–(14) display. And even my remark about varying strength of probabilistic positive relevance carries over. So it seems that the explanatory potential of ranking theory well deserves further exploration; it should be at least qualitatively satisfactory.

However, it seems difficult to establish quantitative relations to the existing experiments, because most of them have a probabilistic bias. For instance, Over et al. (2007, p. 64) note that "one limitation" of a long list of experiments "is that they only concern conditionals about frequency distributions that are assumed to be understood by the participants." Here, the probabilistic bias is explicitly part of those experiments. Over et al. therefore focus on ordinary ('non-basic') conditionals about ordinary affairs not building on frequency data. However, on p. 66 they report to try to avoid all ambiguity by using the question form: "Could you please rate the probability that the statement is true: if …, then …" Again, if you ask for probabilities, you get probabilities.

Sometimes, the probabilistic bias is part of the methodology. Evans and Over (2004, p. 112) report the usual practice to "ask people to make yes or no decisions," for instance

about the acceptability of a conditional inference. And the practice continues to equate the relative frequency of "yes" with the average degree of confidence in the conclusion of the inference. Thereby, however, the experimenter presupposes that these degrees of confidence are probabilities. On the same page, Evans and Over go on reporting that this equation has turned out to be justified, since directly asking people for their degree of confidence would result roughly in the same average. However, now the probabilistic bias is again in the way asking people. Either way, it seems that, if my ranking-theoretic proposals are to have opportunities for experimental check, the experiments should be designed more neutrally.

The ranking-theoretic option (IV.1) may be further differentiated in a way which makes no sense for its probabilistic analog and which may be experimentally significant. For long, I feel justified in calling *A* a *reason for B*, if *A* speaks for *B*, if *A* supports or confirms *B*, if *A* strengthens the belief in *B*—that is, if *A* is positively relevant for *B*, if (IV.1) obtains. (For justification see Spohn [2012].) I chose this label also to indicate the deep philosophical significance of positive relevance. My present point is that there are various kinds of reasons or positive relevance. If *A* is a reason for *B*, it raises the degree of belief in *B*. But from where to where? We can distinguish four cases (with self-explaining labels), and we might have an interest in expressing any of them by using a conditional $A \triangleright B$:

$$\tau(B \mid A) > \tau(B \mid \bar{A}) > 0, \text{ that is, } A \text{ is a supererogatory reason for } B. \tag{IV.1a}$$

$$\tau(B \mid A) > 0 \leq \tau(B \mid \bar{A}), \text{ that is, } A \text{ is a sufficient reason for } B. \tag{IV.1b}$$

$$\tau(B \mid A) \geq 0 > \tau(B \mid \bar{A}), \text{ that is, } A \text{ is a necessary reason for } B. \tag{IV.1c}$$

$$0 > \tau(B \mid A) > \tau(B \mid \bar{A}), \text{ that is, } A \text{ is an insufficient reason for } B. \tag{IV.1d}$$

Thus, for example, *A* is a sufficient reason for *B*, if *B* is believed given *A*, but not given $\bar{A}$. Only kinds (b) and (c) are not disjoint; *A* may be a necessary and sufficient reason for *B*. The same distinctions may be made for negative relevance. In this way, the expressive options for $A \triangleright B$ differentiate further.

Again, the properties of these kinds of reason are not straightforward and not completely known. Some expectations (trained by deductive logic) may be disappointed. For instance, the relation of being a sufficient reason is not transitive. And if *A* and *A′* are sufficient reasons for *B*, $A \cap A'$ need not be! Before turning such disappointments against ranking theory, recall its normative foundations. The illuminating task is always to find hidden premises under which the expectations can be proved to be satisfied. I should also point out that the four kinds of reasons need not be symmetric by themselves; only their disjunction (IV.1) is symmetric. Thus, one should check the extent to which the explanation of the four inferences (MP), (MT), (AC), and (DA) can be maintained after these differentiations.

Also, the logical independence of (IV) from (II) and (III) no longer holds. There are many interesting interactions of (II) and (III) with these types of reason. For instance, if

$A \rhd B$ is a counterfactual expressing the falsity of A and B, (II.3) and (III.3), and more- over expressing not only the Ramsey test (I.1) but also positive relevance (IV.1), then A must be a necessary and sufficient reason for B, that is, (IV.1$b+c$) apply. And so forth. There is no place here to further extend the formal details.

6. What else conditionals might express

It might appear that (I)–(IV) exhaust our expressive options for the generic conditional $A \rhd B$. We can express our attitudes toward A and B by themselves and how we epistemi- cally relate A and B. And since we have refrained from attending to specific degrees of belief, nothing seems left out. Nothing? No, there is at least one further most important class of beliefs that we might express with conditionals. It is clearly suggested by many examples, but if it has received a theoretical treatment, then either vaguely or rather under the heading "counterfactuals" and within the Stalnaker–Lewis tradition, but not within the expressivistic perspective. Let me explain what I have in mind:

We might start with the infamous pair of examples of Quine (1960, p. 222) concerning the Korean war:

$$\text{If Cesar were in command, he would use the atomic bomb.} \qquad (15)$$

$$\text{If Cesar were in command, he would use catapults.} \qquad (16)$$

The pair was designed to demonstrate the hopeless context-dependence and indetermi- nacy of counterfactuals discourse. The case is not so hopeless, though. (15) directs our attention to a certain issue or question: What kind of political leader was Cesar? Violent, audacious, prudent, compromising, appeasing, etc.? (16) raises a different question: What kind of warfare technology was available at Cesar's times?

Formally, a question is represented by a partition of the possibility space W, for instance, a psychological partition each cell of which represents one possible psychological condition of Cesar, or a technological partition, etc. (of course, a rich possibility space has lots of partitions). The question then is: Which cell of the partition is the true one? There must be exactly one that is true. An informative answer need not be maximally specific, though; it has the form: the true cell lies in this or that part of the partition, for instance, in one of the "compromising" cells ("compromising" does not denote a specific cell, but only a coarse set of cells within the fine-grained psychological partition concerning Cesar).

And what do (15) and (16) assert? Clearly, they give an answer to their respective question. (15) says: Cesar is a kind of person who would use the atomic bomb, if in com- mand. And (16) says that Cesar had available such a kind of technology that he would use catapults, if in command. And by uttering (15) and (16) I express the corresponding beliefs.

A bit more formally: Let us abbreviate (15) as $K \rhd A$. With (15), I claim that Cesar is of a certain psychological characteristic C^*. So, I believe C^*, that is, $\tau(C^*) > 0$.

Somehow, C^* is determined with the help of $K \rhd A$. But how? According to what (15) was just observed to assert, C^* is the characteristic such that given this characteristic and Cesar's being in command, he would use the atomic bomb, that is, such that $\tau(A \mid K \cap C^*) > 0$. Is C^* thereby uniquely determined? No. We have to be a bit more careful. What I really express is my belief that Cesar belongs to one of those cells C of the psychological partition for which I believe A given $K \cap C$. That is, I believe in the disjunction of those cells; C^* is the disjunction or union of all those cells C for which $\tau(A \mid K \cap C) > 0$.

We could go through the same exercise with (16) and the appertaining technological partition. This amounts to a clear description of the relevant context-dependence of (15) and (16): The context consists in a certain issue or question or partition, and within that context it is determinate what the conditional assertion claims or expresses. (Moreover, the assertions create their own issues; in this case, their own contexts; they do not merely happen to be located in a certain context. This is a common phenomenon called accommodation in linguistics.)

In any case, the abstract representation of the situation is this: Let $\mathcal{P}$ be a partition of W, that is, a set of mutually disjoint and jointly exhaustive conditions or propositions. We may then use the conditional $A \rhd B$ for expressing the belief that one of all those conditions in $\mathcal{P}$ obtains given which we believe B conditional on A—formally:

$$\tau(C^*) > 0, \text{ that is, } \mathbf{B}(C^*), \text{ where } C^* = \bigcup \{C \in \mathcal{P} \mid \tau(B \mid A \cap C) > 0\}. \qquad \text{(V.1)}$$

The foregoing discussion was guided by the Ramsey test; that is, it focused on the additional conditions C under which the conditional belief in B given A is maintained. However, the discussion of (15) and (16) might have been even more plausible in terms of positive relevance. The conditional $A \rhd B$ might as well be used for expressing the belief that one of all those conditions in P obtains given which A is taken to be positively relevant to B—formally:

$$\tau(C^*) > 0, \text{ that is, } \mathbf{B}(C^*), \text{ where } C^* = \bigcup \{C \in \mathcal{P} \mid \tau(B \mid A \cap C) > \tau(B \mid \bar{A} \cap C)\}. \qquad \text{(VI.1)}$$

Now, expressive options proliferate. Just as (V.1) builds on the Ramsey test (I.1), one might introduce options (V.2) and (V.3) building on (I.2) and (I.3). Similarly, one might define options (VI.2) and (VI.3) paralleling (IV.2) and (IV.3). I am not sure whether all these options can be exemplified. However, it would certainly be worthwhile to differentiate (VI.1) in the same way as we differentiated (IV.1) according to the various kinds of reason or positive relevance.

There are many formal issues then. For instance, does (V.1) entail (I.1), the Ramsey test, that is, the expression of conditional belief *simpliciter*? The answer is: generally no, but under mild conditions yes. So there might be some cause for confusing (I.1) and (V.1). The same questions may be raised with respect to (IV.1) and (VI.1) and their subcategories. Answers to these issues may be found in Spohn (2012, sect. 14.6 and 14.14). It is clear that, if we want to know the logic and the relation of all these expressive

options, we have to study all these issues. Here, it may suffice to have explained this important new type of expressive option.

The most interesting feature of (V.1) and (VI.1) is that according to them conditionals express an *unconditional* belief $\tau(C^*) > 0$, which is truth-evaluable and hence either true or false. This well conforms to our intuition. Look at (15) and (16) again. We may well have a dispute about them, and we all think that this is a factual dispute. What kind of character or political leader was Cesar? Was he really so reckless and aggressive as (15) claims? Which evidence do we find in his biography and his writings for or against (15)? And so forth.

This observation is crucial. In section 2, I strongly emphasized CB*no*TC, the claim that conditional belief has no truth conditions, and thereby motivated my expressivistic strategy. Even if my arguments were good, this may have appeared counterintuitive. Now, we have a partial explanation for this intuition. Maintaining CB*no*TC does not entail that conditionals do not have truth conditions at all; they fail to have them only insofar as they express only conditional belief and nothing else, as they do according to (I) and (IV). However, they may also express other things about conditional belief that are indeed truth-evaluable, and (V) and (VI) show how they might do so.

Still, we must be aware that it depends on various items which truth-evaluable belief is expressed according to (V) and (VI). First, it depends on the partition $\mathcal{P}$, which is somehow contextually given or introduced; we will get rid of this dependence below. And second, it clearly depends on our conditional beliefs; in (V.1) and (VI.1) C^* is defined only relative to our conditional beliefs. Hence, (V) and (VI) still count as also expressing conditional beliefs. And hence, the factuality of our dispute, say, about (15) is precarious. It is so only as long as we share conditional beliefs and thus argue about the same C^*. However, after a heated discussion we may discover that we even disagree in our conditional beliefs and were thus talking at cross-purposes, that is, arguing about different C^*. I think this well agrees with our everyday experience.

7. Counterfactuals and causal conditionals

If (V) and (VI) were only to account for somewhat dubious examples like (15) and (16), then they would not deserve so much attention. However, (V) and (VI) are just the most general context-dependent schemes, referring to a contextually given partition, and (15) and (16) were particularly suited for exemplifying that context-dependence. More important than the general scheme is a special case. That is, the final big claim I want to defend in this article is that all *causal conditionals* instantiate scheme (VI) in a specific, context-independent way.

"Causal conditional" refers to a categorization of conditionals according to the content of the relation between antecedent and consequent. It is, however, strongly correlated with the subjunctive mood and the counterfactual phrasing (although those correlations are never completely reliable). Indeed, the most prominent theory of deterministic causation is the counterfactual one, which started out by defining causal

relations in terms of counterfactuals (cf. Lewis, 1973b). Matters have become more and more complicated, due to numerous causal puzzles like overdetermination and various kinds of preemption (cf. Collins, Hall, & Paul, 2004). There is no point in going into the details, but let me add that the nowadays even more fashionable interventionist account of causation (cf. Pearl, 2000; Woodward, 2003), which works with structural equations or models, explicitly acknowledges to be a variant of the counterfactual approach.

In any case, causal conditionals are very common, and many, if not most counterfactuals are to be interpreted in a causal way. The Ramsey test did not seem adequate for them, since it established only an epistemic and not a causal relation. Therefore, the opinion prevails that they constitute a different type of conditional that requires a different account, say, in terms of the Stalnaker–Lewis semantics or in terms of structural models. Contrary to this opinion, I claim that there is no need to change the framework. Causal conditionals indeed do not follow the Ramsey test (I.1), but they follow scheme (VI.1). Let me explain how they do this.

First, we may assume that the algebra $\mathcal{A}$ over the set W of possibilities is generated by a set $\mathcal{A}^*$ of *simple propositions*, that is, each proposition in $\mathcal{A}$ is a possibly very complex logical combination of propositions in $\mathcal{A}^*$. And we may further assume that each of the simple propositions in $\mathcal{A}^*$ has a fixed temporal location. "It is foggy in Konstanz at November 19, 2012," "I do not sleep well at November 20, 2012," "my flight starts at November 21, 2012, at 6 am": all of these are simple propositions referring to a specific time. Complex propositions need not have a clear temporal location. To which time(s) do conjunctions or disjunctions of my three sample propositions refer?

We may imagine then that the possibilities in W, as far as they can be characterized by $\mathcal{A}$, are maximal consistent conjunctions of simple propositions in $\mathcal{A}^*$ or their negations; each possibility is an entire possible history of the form "first A_1, and then A_2, and then A_3, ...," where A_1, A_2, ... are simple propositions in $\mathcal{A}^*$ or their negations. Those histories need not be complete histories in any absolute sense; they would be so only if the possibilities in W were full possible worlds in the sense of David Lewis. Hence, the histories in W are more or less fine-grained depending on the richness of $\mathcal{A}^*$. These informal descriptions are good enough for our present purpose. Of course, a formal treatment would have to be fully explicit about these algebraic matters, and then issues of granularity would loom large.

Next, we may observe that the antecedent and the consequent of conditionals often refer to simple propositions. The general reason may be logical simplicity, but in the case of causal conditionals the reason is that antecedent and consequent refer to cause and effect, and cause and effect have to have a fixed temporal location. So, a causal conditional has the form $A_t \triangleright B_{t'}$, where A_t refers to t and $B_{t'}$ to t'; for instance, if I had dropped the glass (right now), it would have broken (a second later). We may assume that t' is later than t and neglect here philosophical extravagancies pondering about whether a cause might be simultaneous with or even later than its effect.

Now, what does it mean that A_t is a cause of $B_{t'}$? There is not the slightest hope of adequately dealing here with this issue. (In Spohn [2012], I devote over 100 pages to it.)

Let me only briefly sketch the answer I endorse for more than 30 years: A_t is a *cause* of $B_{t'}$ if A_t is positively relevant to $B_{t'}$ in *some* sense, if A_t makes a contribution to $B_{t'}$, if within the given course of events or on the basis of the actual history H A_t was somehow required to bring about $B_{t'}$, that is, if given the actual history H A_t is positively relevant to $B_{t'}$: if $\tau(B_{t'} \mid A_t \cap H) > \tau(B_{t'} \mid \overline{A_t} \cap H)$.

Thus, very roughly, causes are reasons given the history. The topic "reasons and causes" is an important one in epistemology (as well as in action theory), and for many centuries it has produced profound confusions under varying labels. When I am short-circuiting this issue here, this may be taken as illuminating or as continuing confusion. In any case, my short-circuit has good precedence. To begin with, conditioning on the history H has been first explicitly proposed by Good (1961–63) within statistical attempts at causation, and it is still a virulent idea. The idea is also present in the counterfactual approach. In a way, the crucial issue about counterfactuals is what is cotenable, to be kept fixed, with the counterfactual assumption. And then it is always said that, when the counterfactual is a causal one, it must be understood in a non-backtracking way, that is, as not affecting the past, as leaving the past unchanged and thus as letting the actual history be fixed and given. The very same point is emphasized in the interventionist approach: the idea of an intervention is precisely to keep history fixed and miraculously, as it were, to wiggle only with the cause. So conditioning on the history is a very common idea.

What is unusual about my short-circuit is its appeal to the epistemological notion of a (conditional) reason, thus turning causation into something relative to our epistemic states. However, even this move has good precedence. It is David Hume's move, and it has been one of the most bewildering moves in the entire history of philosophy; it is too appalling to find many followers (though Kant's so-called Copernican revolution of metaphysics builds on it). On the other hand, I cannot find that it has been convincingly rejected; it remains a thorn in the flesh of philosophy. My reason for following Hume is simple and powerful: namely that all objectivist conceptions of causation have been unable to come up with an adequate notion of positive relevance; all in all, all those causal puzzles (overdetermination, preemption, etc.) can be best solved with the epistemic notion of positive relevance (see Spohn, 2012, ch. 14). A supplementary reason is that this epistemic relativization of causation can be undone to a large extent; we need not forswear our objectivistic intuitions (see Spohn, 2012, ch.15).

We should not further digress into philosophical obscurities; let me return to my explanation of causal conditionals. So far, I said that A_t is a cause of $B_{t'}$ iff, given the actual history H, A_t is positively relevant to $B_{t'}$ (according to the epistemic state κ or τ). But we have to be a bit more precise. What is the actual history H? As I have argued several times (cf. Spohn, 2012, sect. 14.4), we should focus on A_t being a *direct* cause of $B_{t'}$ and then take the entire history up to the effect at t' except the cause at t as the relevant history H. Moreover, we should not merely refer to the actual history; we should make explicit that there are many possible histories and that each possibility or possible world has its own history (up to t') and its own causal relations. So let $H^t_{w,t'}$ denote the history or the course of events in possibility w up to time t' with the exception of t. Of course,

how rich $H_{w,t'}^t$ is depends on the richness of the set $\mathcal{A}^*$ of simple propositions originally assumed. So my final explication for the present purposes is

> A_t is a *direct cause* of $B_{t'}$ in the possible world w (relative to the ranking function τ)
>
> if and only if A_t is positively relevant to $B_{t'}$ given $H_{w,t'}^t$, that is, if $\tau(B_{t'} \mid A_t \cap H_{w,t'}^t)$ (17)
>
> $> \tau(B_{t'} \mid \overline{A_t} \cap H_{w,t'}^t).$

Extending this to an account of indirect causation is a very complicated issue (cf. Spohn, 2012, sect. 14.11–13), which need not concern us here.

Now, at long last, we are prepared to address causal conditionals. When I utter the counterfactual "if A_t had not been the case, $B_{t'}$ would not have been the case," intending a causal interpretation, I express my belief that A_t is a (direct) cause of $B_{t'}$. How might this belief be understood in view of (17)? Well, it is the belief that the world or history is such that A_t is a (direct) cause of $B_{t'}$; it is the belief in the truth condition of "A_t is a (direct) cause $B_{t'}$," in the set of a worlds w in which the definiendum or definiens of (17) is satisfied.

Take an example: "If it had not rained, the plants would not have died." That's the same as: "The plants died, because it rained." There are plenty of possible histories in which this is not true. The soil might have been poisoned, some animals might have eaten the plants, some humans might have destroyed them, etc. In asserting one of the two sentences, I express my belief that none of these alternative histories obtains.

Now, we can finally see how causal conditionals fall under the schemes (V) and (VI). Each generic conditional of the form $A_t \triangleright B_{t'}$ can be understood, without any contextual clues, as referring to the set, indeed the partition $\mathcal{H}_{t'}^t$ of possible histories $H_{w,t'}^t$ up to t' with the exception of t. And then we can take the conditional as expressing (V.1) or (VI.1) referring to that specific partition, for example,

$$\tau(H^*) > 0, \text{ where } H^* = \{w \in W \mid \tau(B_{t'} \mid A_t \cap H_{w,t'}^t) > 0\}, \text{ or} \qquad \text{(VII.1)}$$

$$\tau(H^*) > 0, \text{ where } H^* = \{w \in W \mid \tau(B_{t'} \mid A_t \cap H_{w,t'}^t) > \tau(B_{t'} \mid \overline{A_t} \cap H_{w,t'}^t)\}. \qquad \text{(VIII.1)}$$

It is clear that (VII) and (VIII) ramify in the same way (V) and (VI). It is also clear that my explanations concerning causal conditionals aimed at (VIII.1) that includes conditional positive relevance. Perhaps, though, we do not want to include that and only express the belief that history is such that $B_{t'}$ must obtain given A_t (and possibly even not given A_t); then option (VII) is pertinent. Finally, it is clear that, if the conditional $A_t \triangleright B_{t'}$ is to express (VIII.1), then it is to be read as "$B_{t'}$ because A_t." We may also take (VIII.1) as expressing a conditional of the form $\overline{A_t} \triangleright \overline{B_{t'}}$; then, indeed, it is the counterfactual if A_t had not been the case, $B_{t'}$ would not have been the case."

At the beginning of this section, I mentioned that the expressive options just explained have been at least suggested in the literature; the idea is by no means new. It has, for instance, become customary to speak of the basis of a conditional (cf., e.g., Edgington, 1995; p. 283), which is supposed to consist in the facts that make the conditional true or

assertible. When Bennett (2003, ch. 22) gives a central role to the evidence or explanatory bases of conditionals, he has something similar in mind. And when Lewis says that counterfactuals, and thus the similarity ordering on which they ground, supervene on the character of the actual world (cf. Lewis, 1986, p. 22), he refers to such a basis on a grander scale and in a more metaphysical mood. However, at the three places cited and elsewhere, this basis remains dim and does not receive further theoretical treatment. (V)–(VIII) offer subjective counterparts of that basis: the (context- or partition-relative) proposition C^* of (V) and (VI) and the historic proposition H^* of (VII) and (VIII). These propositions well deserve to be called the basis on which the relevant conditional belief is held, and their explication opens the way to a rigorous and detailed study of that basis of conditionals.

This concludes my list of expressive options for conditionals. It goes far beyond the Ramsey test, and as my many examples displayed, most of those options are required for accounting for the phenomena. Let me demonstrate the power of my approach with a final example:

Adams (1970) has introduced the famous pair:

$$\text{If Oswald did not kill Kennedy, someone else did.} \tag{18}$$

$$\text{If Oswald had not killed Kennedy, no one else would have}$$
$$\text{(and Kennedy would have been alive, for a while).} \tag{19}$$

Let O = Oswald killed Kennedy" and S = "someone else killed Kennedy." Then (18) may be abbreviated as $\bar{O} \vartriangleright_1 S$ and (19) as $\bar{O} \vartriangleright_2 \bar{S}$. The thrust of the pair is obvious and powerful: Both (18) and (19) are true or at least clearly acceptable. Hence, $\vartriangleright_1$ and $\vartriangleright_2$ must be two different kinds of conditionals. Thus, very different theories evolved for what is usually classified as indicative and subjunctive conditionals—a most dramatic effect of the pair (18) and (19).

Stalnaker et al. (1975) tried to preserve unity by proposing that (18) and (19) involve a context shift so that $\vartriangleright_1$ and $\vartriangleright_2$ are to be interpreted in two different contexts. (19) can be uttered only in a context where O is assumed to be true, whereas (18) makes sense only in a context where O is treated as open. And then, Stalnaker goes on to explain how each conditional may be acceptable in its own context. However, I do not find the claim about (18) convincing. It is perfectly acceptable to say within one context: "I do believe that Oswald killed Kennedy; we all do. But if he did not, someone else must have killed Kennedy." Indicative conditionals need not be open conditionals.

Bennett (2003) seeks a unified treatment in identifying the varying explanatory schemes underlying the various conditionals. Edgington (1995, p. 215; 2008, pp. 4ff.) attempts to preserve unity within a probabilistic framework. There is no place to discuss all of these approaches, though my article contains hints why I tend to be critical of them. In any case, here is a perfectly straightforward account of this example within my framework, which neither involves context shift nor different theories of conditionals, but

only one ranking function and two expressive options. First, it is clear that we believe O and reject S. Hence, our two-sided ranking function τ is such that

$$\tau(O) > 0 \text{ and } \tau(\bar{S}) > 0. \tag{20}$$

Let us not complicate our business and disregard relevance considerations. Then (18), or $\triangleright_1$, is most plausibly interpreted according to the Ramsey test (I.1) and thus expresses:

$$\tau(S \mid \bar{O}) > 0. \tag{21}$$

It does not merely express, though, a belief in the material implication $\bar{O} \rightarrow S$. This is so, because the antecedent is believed to be false according to (19), and hence the condition for the equivalence in (11) is not satisfied.

My further suggestion was that counterfactuals are interpreted according to scheme (VII.1) (again neglecting relevance considerations). If we apply this to (19), or $\triangleright_2$, (19) says that history is such (= H^*) that, if $\bar{O}$ and H^*, then $\bar{S}$. So, H^* agrees with the Warren report confirming that Oswald was a single assassin, and $\overline{H^*}$ would include a conspiracy theory. Thus, (19) expresses

$$\tau(H^*) > 0 \text{ and } \tau(\bar{S} \mid \bar{O} \cap H^*) > 0. \tag{22}$$

All these conditional and unconditional beliefs (20)–(22) perfectly go together in one consistent doxastic state—in fact ours—and thus are well expressed in one and the same context by (18) and (19). For proof, look at the following Table 2, which gives a plausible version of our negative ranking function κ:

Since the middle left upper box contains the only 0, κ believes O, $\bar{S}$, and H^*; this accounts for (20) and one half of (22). Moreover, we have $\kappa(\bar{O}) = \min \{6, 4, 3, 4\} = 3, \kappa(\bar{O} \cap \bar{S}) = 4$, and thus $\kappa(\bar{S} \mid \bar{O}) = 1$; that is, S is believed given $\bar{O}$, as required by (21). Finally, the two upper boxes to the right say that $\kappa(S \mid \bar{O} \cap H^*) = 6 - 4 = 2$; that is, S is disbelieved, and $\bar{S}$ believed, under this condition, as required by the other half of (22).

This consistency proof may be a bit surprising. One might have expected that if one holds a certain (conditional) belief under some (additional) condition H^* and moreover believes in that condition H^*—as (22) asserts—then one should also hold that (conditional) belief without that (additional) condition H^*—in contradiction to (21). However, this expectation is fallacious, and the table shows how it can be wrong. The expectation

Table 2
Oswald–Kennedy

κ	$O \cap S$	$O \cap \bar{S}$	$\bar{O} \cap \bar{S}$	$\bar{O} \cap \bar{S}$
H^*	2	0	6	4
$\overline{H^*}$	1	2	3	4

is correct only under additional assumptions, which cannot be satisfied in this example (cf. Spohn, 2012, Theorems 14.14 and 14.81).

8. Concluding remarks

This article attempted to demonstrate a unified expressionist framework for interpreting our rich assertive conditional idiom. By providing various expressive alternatives to the Ramsey test, it offered rich interpretive options within the unified scheme. Indeed, I wanted to raise some hope for systematization. After all, it seems pretty hopeless to directly systematize the linguistic data. One problem is that focusing on these data is not the best heuristics for theoretical insight, and the other problem is the enormous interaction with all kinds of pragmatic processes. Therefore, it looks advisable first to start with a systematization of the epistemic states conditionals might express. Here, this article attempts to make progress. I do not claim that my epistemic systematization is complete. I have only listed all the expressive options I could think of. I am curious to learn whether there are further ideas not covered by my list.

Let me finally return to my fundamental claim CB*no*TC that conditional belief has no truth conditions. I so strongly emphasized it in order to promote my expressivistic perspective which is, I think, the only one offering the attempted systematization. Still, my basic strategy may have appeared appalling. Even if CB*no*TC is true in general, does our conditional idiom really only express conditional belief and is thus not truth-evaluable? No, this deeply runs counter to our intuition and experience.

At the end of such a long study, I cannot do any justice to such a deep and fundamental issue. I only want to point out that I have made significant room for a truth-conditional perspective. True, the basic options (I) and (IV) did not do so. But the somewhat trivial, partial options (II) and (III) did. And so did the important options (V) and (VI), because they expressed truth-evaluable unconditional belief. This expression was doubly relative, though. It first depended on which partition $\mathcal{P}$ was contextually alluded to. And second, the unconditional belief expressed depended on the conditional beliefs held. Still, there is progress in (V) and (VI); insofar as at least the relevant conditional beliefs are intersubjectively agreed upon, dispute about conditionals is purely factual dispute about unconditional beliefs; within these confines (V) and (VI) allows us to assign truth conditions to conditionals.

The situation further improves with (VII) and (VIII). As explained, causal conditionals provide the relevant partition by themselves; no such context-dependence intrudes. Moreover, one might argue that the intersubjective agreement in conditional beliefs alluded to may be more easily reached in the case of the conditional beliefs referred to in (VII) and (VIII). More important, though, there are specific objectivization strategies for precisely the conditionals of that specific form; CB*no*TC holds generally, but not universally. For this claim, I can only refer to Spohn (2012, sect. 15.4–5). The upshot is that according to (VII) and (VIII), conditionals are truth-evaluable in a full sense; they express a truth-evaluable unconditional belief not only relative to subjective or hopefully intersubjectively shared conditional beliefs but also relative to conditional beliefs which in turn are

truth-evaluable. This outlook deserves to be fully explained. Here, it must remain a mere outlook.

Acknowledgments

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