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EPSA Epistemology and Methodology of Science

Launch of the European Philosophy of Science Association

Springer

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ISBN 978-90-481-3262-1 e-ISBN 978-90-481-3263-8
DOI 10.1007/978-90-481-3263-8
Springer Dordrecht Heidelberg London New York

Library of Congress Control Number: 2009941460

Cover design: Boekhorst Design b.v.

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Contents

Introduction ix

1 **Naturalism and the Scientific Status of the Social Sciences** 1
Daniel Andler

2 **Reconsidering Gilbert's Account of Social Norms** 13
Caroline M. Baumann

3 **Theories for Use: On the Bearing of Basic Science on Practical Problems** 23
Martin Carrier

4 **Structural Realism as a Form of Humility** 35
Angelo Cei

5 **Approaching the Truth via Belief Change in Propositional Languages** 47
Gustavo Cevolani and Francesco Calandra

6 **Can Graphical Causal Inference Be Extended to Nonlinear Settings?** 63
Nadine Chlaß and Alessio Moneta

7 **Towards a Grammar of Bayesian Confirmation** 73
Vincenzo Crupi, Roberto Festa, and Carlo Buttasi

8 **Epistemic Accuracy and Subjective Probability** 95
Marcello D'Agostino and Corrado Sinigaglia

9 **Interpretation in the Natural Sciences** 107
Jan Faye

Gigerenzer G et al. (1999) Simple heuristics that make us smart. Oxford University Press, New York
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Norton J (2003) A material theory of induction. Philos Sci 70:647–670
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Chapter 24
Multiple Contraction Revisited

Wolfgang Spohn

24.1 Introduction

Belief revision theory studies three kinds of doxastic movements: expansions, revisions, and contractions. Expansions and revisions are about learning or acquiring new beliefs. Expansion is the unproblematic case where the new belief is consistent with the old ones and can hence be added without further ado. Revision is the problematic case where the new belief is inconsistent with the old ones and can hence be accepted only when some of the old beliefs are given up; the problem is to find rules for this process. Contractions are directly about giving up beliefs without adding new ones. If we require beliefs to be deductively closed, this is problematic, since we cannot simply delete the belief in question; the remaining beliefs would entail it in turn. So, again the problem is to find rules for this process.

There is an argument over the priority of these doxastic movements. As I have presented them, revision seemed to be a composite process, a contraction followed by an expansion. This view is championed by Isaac Levi, e.g., in Levi (2004). Others wonder how there can be genuine contractions; even for giving up beliefs you need to get a reason, i.e., to learn something. There is no need to decide the argument. I think there are good reasons for taking revisions and contractions to be on a par, firmly connected by Levi's and by Harper's identity (cf., e.g., Gärdenfors 1988, sect. 3.6). This paper will be mainly about contractions and mention revisions only supplementarily.

I believe that the three movements are dealt with by the well-known AGM theory (Alchourrón et al. 1985; Gärdenfors 1988) in a completely satisfactory way; I shall state my main reason below. Of course, there is a big argument over the adequacy of the AGM postulates for revision and contraction; see, e.g., the many alternative postulates in Rott (2001, ch. 4). However, let us be content here with the standard AGM theory.

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M. Suárez et al. (eds.), *EPSA Epistemology and Methodology of Science: Launch of the European Philosophy of Science Association*, DOI 10.1007/978-90-481-3263-8_24,

Still, there are problems the standard theory cannot cope with. One kind of problem concerns the rules for making several movements. I state this so vaguely, because the problem takes at least two forms. The main form is the problem of *iteration* first raised in Spohn (1983, ch. 5). How should you revise or contract your beliefs several times? What are the rules for doing so? Some iterative postulates seem accepted, some are contested (see the overview in Rott 2009), and the issue is still very much under dispute.

A minor form is a problem first discussed by Fuhrmann (1988); it is about *multiple contraction*, as he called it. There the issue is to give up several beliefs not one after the other, but at once. This is a relevant issue. Suppose you read a newspaper article, and you accept all details of its surprising story. In the next issue, the journalist apologizes; the article was a 1st April's joke, a make-believe from the beginning to the end. In this way, it is not uncommon that several of your beliefs at once turn our not to be maintainable.

In fact, Fuhrmann and Hansson (1994) distinguish two forms of multiple contraction. The contraction of a set $\{A_1, \ldots, A_n\}$ of beliefs may take the form of *choice contraction* where you are requested to give up at least one of the beliefs $A_1, \ldots, A_n$. This form is probably uninteresting; in any case it has a simple solution: the request is the same as that for a single contraction of the conjunction of $A_1, \ldots, A_n$; if you give up the conjunction, you have to give up at least one of the conjuncts. Hence, this case is covered by standard AGM contraction.

The other form is what they call *package contraction* where you are indeed asked to give up all the beliefs $A_1, \ldots, A_n$, as it was in my example above. Now the answer is not obvious at all; we shall see below why the obvious attempts are inadequate. In fact, as far as I know, the problem has not found a satisfactory solution. Fuhrmann and Hansson (1994) propose some axioms partially characterizing package contraction, but they are quite tentative about these axioms and apparently not satisfied. Fuhrmann (1997) does not get beyond these axioms, and Hansson (1999, sect. 2.16) acquiesces in a weak axiomatization, which, however, he is able to show to be complete relative to the weak semantics he proposes. As far as I know, the problem became neglected afterwards. The goal of the paper is to present a complete and satisfactory solution.

As observed by Hansson (1999, sect. 3.17) there is the parallel problem of *multiple revision* by a set $\{A_1, \ldots, A_n\}$ of beliefs. In *package revision*, you are asked to accept all of the new beliefs $A_1, \ldots, A_n$. This is obviously the same as accepting their conjunction, and thus the case reduces to single AGM revision. In *choice revision*, you are requested to accept at least one of those beliefs, and again there is no obvious answer. (Note that you may accept their disjunction without accepting any of the disjuncts; so accepting their disjunction is no way to meet to request.) The problem is as difficult as that of package contraction. I assume it can be solved by similar means. However, I shall not pursue this case here, since it seems artificial and without natural applications, unlike the case of package contraction.

The basis of my solution is ranking theory, as I have developed it in Spohn (1983, 1988); see also my survey in Spohn (2009). It proposes a general dynamics of belief which comprises expansions, revisions, and contractions as special cases, which is iterable, and which hence solves the problem of iterated belief revision and contraction. Hild and Spohn (2008) show which set of laws of iterated contraction is entailed by ranking theory and proves its completeness. Ranking theory also provides a plausible model of multiple contraction, as I hope to show below; the behavior of multiple contraction entailed by it will turn out to be quite simple. The issue is much less involved than the problem of iteration.

The plan of the paper is straightforward. In Section 24.2 I shall explain the problem of package contraction in formal detail. Section 24.3 will introduce the ranking theoretic material as far as needed. Section 24.4, finally, will present the ranking theoretic account of package contraction.

24.2 The Problem of Multiple Contraction

Let me first recall the AGM account of contraction, in an equivalent form. The standard way is to represent beliefs, or rather their contents, by sentences, presumably because one wanted to do logic. However, the formalism is much simpler when beliefs are represented by propositions; one need not worry then about logically equivalent sentences. This is the way I always preferred.

Hence, let W be a non-empty set of possibilities or possible worlds, and $\mathcal{A}$ be an algebra of subsets of W. The members of $\mathcal{A}$ are *propositions*. For the sake of simplicity, I shall assume $\mathcal{A}$ to be finite; but nothing depends on this. The first notion we need is that of a belief set:

Definition 1. $\mathcal{K}$ is a *belief set* iff $\mathcal{K}$ is a proper filter in $\mathcal{A}$, i.e., iff, given the finiteness of $\mathcal{A}$, there is a proposition $C(\mathcal{K}) \neq \emptyset$, the *core* of $\mathcal{K}$, such that $\mathcal{K} = \{A \in \mathcal{A} | C(\mathcal{K}) \subseteq A\}$.

By assuming $C(\mathcal{K})$ to be non-empty, I exclude inconsistent belief sets right away. Deductive closure of belief sets is built into definition 1 and into the propositional approach.

In this approach the AGM theory of contraction looks thus:

Definition 2. $\div$ is a *single contraction* for the belief set $\mathcal{K}$ iff $\div$ is a function from $\mathcal{A} - \{W\}$ into the set of belief sets such that:

(a) $\mathcal{K} \div A \subseteq \mathcal{K}$ [Inclusion]
(b) if $A \notin \mathcal{K}$, then $\mathcal{K} \div A = \mathcal{K}$ [Vacuity]
(c) $A \notin \mathcal{K} \div A$ [Success]
(d) if $B \in \mathcal{K}$, then $A \rightarrow B \in \mathcal{K} \div A$ [Recovery]
(where $A \rightarrow B = \bar{A} \cup B$ is the set-theoretic analogue to material implication)
(e) $\mathcal{K} \div A \cap \mathcal{K} \div B \subseteq \mathcal{K} \div (A \cap B)$ [Intersection]
(f) if $A \notin \mathcal{K} \div (A \cap B)$, then $\mathcal{K} \div (A \cap B) \subseteq \mathcal{K} \div A$ [Conjunction]

These are the set-theoretic translations of the AGM contraction postulates. The closure postulate is part of my characterization of the range of $\div$, and the extensionality postulate is implicit in the propositional approach. The necessary proposition W cannot be given up and is hence excluded from the domain of $\div$; one might certainly acknowledge more necessary propositions, as AGM actually do. AGM also assume the contraction function to work for all belief sets; here, it suffices to define it only for a given belief set $\mathcal{K}$.

So much is settled. Now, let $\mathcal{B}$ be any set of non-necessary propositions in $\mathcal{A} - \{W\}$, and let $\mathcal{K} \div [\mathcal{B}]$ denote multiple contraction in the package sense. The intended meaning is clear; all the propositions in $\mathcal{B}$, insofar they are believed in $\mathcal{K}$, have to be given up. How and according to which rules is package contraction to be carried out? Is it definable in terms of single contractions?

In order to develop a sense for the difficulty of the problem, let us look at the simplest genuine case where $\mathcal{B} = \{A, B\}$, i.e., at the contraction $\mathcal{K} \div [A, B]$ of two propositions A and B from $\mathcal{K}$. It may obviously not be explained as $\mathcal{K} \div A \cap B$. To contract by the conjunction guarantees only that at least one of the conjuncts has to go; but the other may be retained, and then the package reduction would be unsuccessful. Success, i.e. $(\mathcal{K} \div [\mathcal{B}]) \cap \mathcal{B} = \varnothing$, is, no doubt, a basic requirement. In other words: it would be wrong to equate package contraction with choice contraction. They agree only in the degenerate case of contraction by a singleton.

Nor may package contraction $\mathcal{K} \div [A, B]$ be explained as $\mathcal{K} \div (A \cup B)$. This would guarantee success; if the disjunction has to give way, the disjuncts have to do so, too. However, the proposal is clearly too strong. One may well give up both disjuncts while retaining the disjunction; in any case, this should not be excluded.

Package contraction must also be distinguished from iterated contraction. The easiest way to see this is that iterated contractions need not commute; we may have $(\mathcal{K} \div A) \div B \neq (\mathcal{K} \div B) \div A$. When one asks in such a case with which of the two terms $\mathcal{K} \div [A, B]$ should be identified, the obvious answer is: with none. There is no such asymmetry in the idea of package contraction.

Hansson (1993) gives a nice example in which commutativity of iterated contractions intuitively fails.

> In the ongoing conflict between India and Pakistan, troops have been sent to the border from both sides. A friend has told me that an agreement has been reached between the two countries to withdraw the troops. I believe in this. I also believe that each of the two governments will withdraw its troops if there is such an agreement, but for some reason my belief in the compliance of the Pakistan government is stronger than my belief in the compliance of the Indian government.
>
> Let s denote that there is an agreement to withdraw troops on both sides, p that the Pakistan government is going to withdraw its troops and q that the Indian government is going to withdraw its troops. Then (the relevant part of) my belief base is $\{s, s \to p, s \to q\}$.
>
> **Case 1.** The morning news on the radio contains one single sentence on the conflict: "The Indian Prime Minister has told journalists that India has not yet decided whether or not to withdraw its troops from the Pakistan border." When contracting q, I have to choose between retaining s and $s \to q$. Since my belief in the latter is weaker, I let it go, and the resulting belief base is $\{s, s \to p\}$.
>
> The evening news also contains one single sentence on the conflict, namely: "The Pakistan government has officially denied that any decision has been taken on the possible withdrawal of Pakistan troops from the Indian border." I now have to contract p. This involves a choice between retaining s and retaining $s \to p$. Because of my strong belief in the latter, I keep it, and the resulting belief base is $\{s \to p\}$.
>
> **Case 2.** The contents of the morning and evening news are interchanged.
>
> In the morning, when contracting p from the original belief base $\{s, s \to p, s \to q\}$, I retain $s \to p$ rather than s, because of the strength of my belief in the former. The resulting belief base is $\{s \to p, s \to q\}$. The contraction by q that takes place in the evening leaves this set unchanged. (Hansson 1993, p 648)

Fuhrmann and Hansson (1994) discuss a final option, namely that $\mathcal{K} \div [A, B] = \mathcal{K} \div A \cap \mathcal{K} \div B$. Then, even though package contraction is not an ordinary contraction, it might be explained in the latter terms and thus could be reduced away as an independent phenomenon. However, they are not happy with that option, either, because they believe to see its incompatibility with the approach they chose instead (cf. Fuhrmann and Hansson (1994), p 62). I believe they were mistaken, as we shall soon see. On the other hand, it is intuitively not fully perspicuous that this should be the right explanation. So, one must look for another approach, anyway.

The only approach left for Fuhrmann and Hansson (1994) is the axiomatic one: if we cannot define package contraction, we can at least try to characterize it. And so they start appropriately generalizing the AGM postulates. This works convincingly for Inclusion, Vacuity, Success, and Recovery, and they even produce representation results for their generalization (see their theorem 9 on p 59). However, they are not sure what to do with Intersection and Conjunction; Sven Ove Hansson told me that he no longer believes in the proposals made there on p 56.

Instead, in Hansson (1999, sect. 2.16) he offers a different axiomatic characterization in the AGM style. That is, he generalizes the notion of a selection function so basic to the AGM approach to the notion of what he calls a package selection function and then proposes to define package contraction as a partial meet package contraction relative to such a package selection function. The relevant axiomatization contains adaptations of Inclusion, Vacuity, and Success and a strengthening of Recovery called P-relevance; it does not contain, however, anything corresponding to Intersection and Conjunction that are so important to single contractions. Since, no progress seems to have made on this point.

24.3 Required Basics of Ranking Theory

I believe that ranking theory can help here and provide a plausible account of multiple contraction in the package sense. In order to present it, I have to develop the relevant portion of ranking theory. The basic notion is this:

Definition 3. κ is a *ranking function* for $\mathcal{A}$ iff κ is a function from $\mathcal{A}$ into $\mathbf{N}' = \mathbf{N} \cup \{\infty\}$ such that

(a) $\kappa(W) = 0$ and $\kappa(\varnothing) = \infty$

(b) $\kappa(A \cup B) = \min\{\kappa(A), \kappa(B)\}$ [the law of disjunction].

Ranks are to be understood as degrees of disbelief. $\kappa(A) = 0$ says that A is not disbelieved; $\kappa(A) > 0$ says that A is disbelieved (to some degree); hence $\kappa(\bar{A}) > 0$ expresses belief in A. (a) and (b) entail

(c) either $\kappa(A) = 0$ or $\kappa(\bar{A}) = 0$ or both [the law of negation].

Of course, both may be 0, in which case κ is neutral or unopinionated about A. The law of negation and the law disjunction ensure that $\{A|\kappa(\bar{A}) > 0\}$ is indeed a consistent and deductively closed belief set. Let us denote this belief set by $\mathcal{K}(\kappa)$.

A main reason for giving the basic role to disbelief rather than belief is the following definition of conditional ranks (that would be less perspicuous in terms of degrees of belief):

Definition 4. If $\kappa(A) < \infty$, the *rank* of $B \in \mathcal{A}$ *given* or *conditional on* A is defined as $\kappa(B|A) = \kappa(A \cap B) - \kappa(A)$.

Equivalently, we have

(d) $\kappa(A \cap B) = \kappa(A) + \kappa(B|A)$ [the law of conjunction]

that says that your degree of disbelief in A-and-B is given by your degree of disbelief in A plus the additional degree of disbelief in B given A. This is intuitively most plausible.

The dynamics of subjective probabilities is stated in various conditionalization rules in terms of conditional probabilities. Likewise, the definition of conditional ranks helps us stating a dynamics of belief and disbelief. The idea is not simply that by learning A you move to the ranks given A. This would assume that you learn A with maximal certainty that can never be undone. The idea is rather that you learn A with some, but not necessarily maximal firmness, as Jeffrey conditionalization (Jeffrey 1965, ch. 11) proposes in the probabilistic case. If, following Jeffrey, we assume, moreover, that by learning A your conditional ranks given A and given $\bar{A}$ do not change, we are able to state our first rule of belief change:

Definition 5. Let $\kappa(A) < \infty$ and $n \in \mathbf{N}'$. Then, the *A,n-conditionalization* $\kappa_{A,n}$ of the ranking function κ is defined by $\kappa_{A,n}(B) = \min\{\kappa(B|A), n + \kappa(B|\bar{A})\}$.

It is easily checked that this preserves conditional ranks given A and given $\bar{A}$ and that $\kappa_{A,n}(A) = 0$ and $\kappa_{A,n}(\bar{A}) = n$ so that A is believed with firmness n in $\kappa_{A,n}$. It is also clear that only $\kappa_{A,n}$ has these two properties. So, the idea is that, rationally, your posterior belief state is always some *A,n-conditionalization* of your prior belief state. Note that this form of conditionalization can be arbitrarily iterated. Thus ranking theory has no problem with iterated belief change.

We shall need a slight generalization of definition 5. Jeffrey has already envisaged the possibility that experience or learning induces you to have changed probabilities for several propositions. We can copy this in ranking theory. Let $\mathcal{E}$ be any (experiential) subalgebra of $\mathcal{A}$, and let λ be any ranking function for $\mathcal{E}$ expressing your experientially acquired degrees of disbelief for $\mathcal{E}$. Then we have:

Definition 6. The *$\mathcal{E},\lambda$-conditionalization* $\kappa_{\mathcal{E},\lambda}$ of κ is defined by $\kappa_{\mathcal{E},\lambda}(B) = \min\{\kappa(B|A) + \lambda(A)|A$ is an atom of $\mathcal{E}\}$.

(Here, the atoms of $\mathcal{E}$ are the logically strongest, i.e., smallest consistent propositions in $\mathcal{E}$; they partition $\mathcal{E}$.) This entails that $\kappa_{\mathcal{E},\lambda}(A) = \lambda(A)$ for all $A \in \mathcal{E}$ and, again, that all conditional ranks given any atom of $\mathcal{E}$ are preserved; they do not change just by learning news about $\mathcal{E}$.

All these definitions are intuitively motivated and well entrenched in ranking theory, a point that can be hardly conveyed in such a brief sketch. For details, I refer the interested reader to the survey in Spohn (2009).

A,n-conditionalization generalizes expansion, revision, and contraction. If you are initially unopinionated about A, i.e., $\kappa(A) = \kappa(\bar{A}) = 0$, then for any $n > 0$ the A,n-conditionalization of κ obviously amounts to an expansion of your initial belief set $\mathcal{K}(\kappa)$ by A (and indeed for any $n > 0$ to the same expansion). If you initially disbelieve A, i.e., $\kappa(A) > 0$, then for any $n > 0$ the A,n-conditionalization of κ amounts to a revision of $\mathcal{K}(\kappa)$ by A (and again for any $n > 0$ to the same revision). The A,0-conditionalization of your initial κ makes you unopinionated about A. If you initially believe A, this change is a contraction by A; if you initially believe $\bar{A}$, it is a contraction by $\bar{A}$. Thus, we may define:

Definition 7. If $\kappa(\bar{A}) < \infty$, then the *contraction* $\kappa_{\div A}$ of κ by A is given by $\kappa_{\div A} = \kappa$, if $\kappa(\bar{A}) = 0$, and $\kappa_{\div A} = \kappa_{A,0}$, if $\kappa(\bar{A}) > 0$. $\div$ is a *ranking contraction* for $\mathcal{K}$ iff for some ranking function κ $\mathcal{K} = \mathcal{K}(\kappa)$ and $\mathcal{K} \div A = \mathcal{K}(\kappa_{\div A})$ for all $A \in \mathcal{A} - \{W\}$.

As observed in Spohn (1988, footnote 20), expansion, revision, and contraction thus explained in a ranking theoretic way satisfy exactly the AGM postulates. In particular, $\div$ is a single contraction according to definition 2 if and only if it is a ranking contraction. Since I am fond of ranking theory, this is my main reason for accepting all the AGM postulates.

Conditionalization indeed generalizes these forms of belief change in several ways. One aspect is that the three forms do not exhaust all ways of A,n-conditionalization; for instance, A may be initially believed and thus the belief in it only be strengthened or weakened by learning. The other aspect is iteration. definition 7 can obviously be iterated and thus provide a model of iterated contraction. Hild and Spohn (2008) give a complete set of postulates governing iterated contraction thus construed. So, let us see how these ideas may help with our present problem of multiple contraction.

24.4 A Ranking Theoretic Account of Multiple Contraction

We start with a ranking function κ for $\mathcal{A}$ and a set $\mathcal{B} \subseteq \mathcal{A}$ of propositions, and we ask how to change κ and its associated belief set $\mathcal{K}(\kappa)$ so that none of the propositions in $\mathcal{B}$ is still believed in κ. It is clear that we may restrict attention to $\mathcal{B}' = \mathcal{B} \cap \mathcal{K}(\kappa)$, the propositions in $\mathcal{B}$ believed in κ, since contraction is vacuous for the other propositions in $\mathcal{B}$. Section 24.2 has shown, moreover, that we may have to deal with logical combinations of propositions in $\mathcal{B}'$. Let us focus hence on the algebra $\mathcal{B}^*$ of propositions generated by $\mathcal{B}'$. There is no reason why propositions outside $\mathcal{B}^*$ should become relevant.

Now we should proceed as follows. We should start with contracting $\bigcap \mathcal{B}'$, the strongest proposition in $\mathcal{B}^*$ believed; it must be given up in any case. This is the same as choice contraction by $\mathcal{B}'$, and we have noted that it removes at least some beliefs in $\mathcal{B}'$. If we are lucky, it even removes all beliefs in $\mathcal{B}'$; then we are done with the package reduction. This would be exceptional, though. Normally, we shall have moved from the prior belief set $\mathcal{K}(\kappa) = \mathcal{K}_0$ to a smaller belief set $\mathcal{K}_1 \subseteq \mathcal{K}_0$ which still believes some propositions in $\mathcal{B}'$. So, in a second step, we again proceed as cautiously as possible and contract the strongest proposition in $\mathcal{B}^*$ still believed, i.e., $\bigcap \{A \in \mathcal{B}^* | A \in \mathcal{K}_1\}$. Possibly, package contraction is now completed. If not, we have arrived at a belief set $\mathcal{K}_2 \subseteq \mathcal{K}_1$ that still holds on to some other beliefs in $\mathcal{B}'$. Then we add a third step, and so on until all beliefs in $\mathcal{B}'$ are deleted. This procedure must stop after finitely many steps.

The conception behind this procedure is the same as in ordinary contraction of a single proposition: change as little as possible till the contraction is successful, where minimal change translates here into giving up the weakest beliefs, the negations of which receive the lowest positive ranks.

Let us cast this into formal definition: Let $\{E_0, \ldots, E_k\}$ be the set of atoms of $\mathcal{B}^*$. Let $E_0 = \bigcap \mathcal{B}'$. So, $\kappa(E_0) = 0$ and $\kappa(E_i) > 0$ for $i = 1, \ldots, k$. Hence, the first contraction informally described above is an E_0, 0-conditionalization. Thereby, some further atoms receive rank 0, say E_1 and E_2, so that $E_3 \cup \cdots \cup E_k$ is still disbelieved. The second contraction outlined above then is an $E_1 \cup E_2$, 0-conditionalization. And so on. Let $R = \{\kappa(E) | E$ is an atom of $\mathcal{B}^*\}$ be the set of ranks occupied by the atoms of $\mathcal{B}^*$. Let $m = \min\{n \in R | \bigcup \{E | E$ is an atom of $\mathcal{B}^*$ and $\kappa(E) > n\} \subseteq \bar{A}$ for all $A \in \mathcal{B}'\}$. If we set only all atoms E with $\kappa(E) < m$ to 0, contraction of the whole of $\mathcal{B}'$ is not yet completed; if we set all atoms E with $\kappa(E) \leq m$ to 0, contraction of $\mathcal{B}'$ is successful, and if we set more atoms to 0, we have contracted more than necessary. So, m is the margin where our contraction procedure stops. Hence, define the ranking function λ on $\mathcal{B}^*$ by $\lambda(E) = 0$ if $\kappa(E) \leq m$ and $\lambda(E) = \kappa(E) - m$ if $\kappa(E) > m$ (and $\lambda(A)$ for non-atoms A of $\mathcal{B}^*$ according to the law of disjunction). My proposal for explicating package contraction thus results in the following

Definition 8. Let κ, $\mathcal{B}$, $\mathcal{B}^*$, and λ be as just explained. Then, the *package contraction* $\kappa_{\div[\mathcal{B}]}$ of κ by $\mathcal{B}$ is the $\mathcal{B}^*, \lambda$-conditionalization of κ. And the *package contraction* $\mathcal{K}(\kappa) \div [\mathcal{B}]$ *of the belief set* $\mathcal{K}(\kappa)$ of κ by $\mathcal{B}$ is the belief set of $\mathcal{K}(\kappa_{\div[\mathcal{B}]})$.

In this way, package contraction turns out as a special case of generalized conditionalization specified in definition 6. Note that my intuitive explanation of package contraction was in terms of successive contractions; but in order to describe the result in one step we require the expressive power of generalized conditionalization.

It easily checked that this model of package contraction satisfies all the postulates endorsed by Fuhrmann and Hansson (1994, pp 51–54). If we accept the explication, we can immediately complete their theory of package contraction. First, it is obvious from the construction above that:

(1) if $\mathcal{B} \subseteq \mathcal{C}$, then $\mathcal{K}(\kappa) \div [\mathcal{C}] \subseteq \mathcal{K}(\kappa) \div [\mathcal{B}]$.

A fortiori, we have

(2) $\mathcal{K}(\kappa) \div [\mathcal{B}] \cap \mathcal{K}(\kappa) \div [\mathcal{C}] \subseteq \mathcal{K}(\kappa) \div [\mathcal{B} \cap \mathcal{C}]$,

which translates into the ranking framework what Fuhrmann and Hansson (1994, p 56) propose as generalization of Intersection. Moreover, it is obvious from our construction that:

(3) if for all $B \in \mathcal{B}$ $B \notin \mathcal{K}(\kappa) \div [\mathcal{C}]$, then $\mathcal{K}(\kappa) \div [\mathcal{C}] \subseteq \mathcal{K}(\kappa) \div [\mathcal{B} \cup \mathcal{C}]$.

If by contracting $\mathcal{C}$ the whole of $\mathcal{B}$ is contracted as well, our iterative procedure for contracting $\mathcal{B} \bigcup \mathcal{C}$ must stop at the same point as that for contracting $\mathcal{C}$. (3) is what Fuhrmann and Hansson (1994, p 56) offer as generalization of Conjunction. Thus, their tentative proposals are in fact confirmed by our model.

Indeed, the most illuminating result concerning our explication is:

(4) $\mathcal{K}(\kappa) \div [A_1, \ldots, A_n] = \mathcal{K}(\kappa) \div A_1 \cap \cdots \cap \mathcal{K}(\kappa) \div A_n$.

Proof. (1) entails that $\mathcal{K}(\kappa) \div [A_1, \ldots, A_n] \subseteq \mathcal{K}(\kappa) \div A_i$ for $i = 1, \ldots, n$. This proves one direction. Reversely, assume that $\mathcal{K}(\kappa) \div A_1 \cap \cdots \cap \mathcal{K}(\kappa) \div A_{i-1} \subseteq \mathcal{K}(\kappa) \div [A_1, \ldots, A_{i-1}]$. If $A_i \notin \mathcal{K}(\kappa) \div [A_1, \ldots, A_{i-1}]$, then $\mathcal{K}(\kappa) \div [A_1, \ldots, A_i] = \mathcal{K}(\kappa) \div [A_1, \ldots, A_{i-1}]$ and there is nothing more to show. If $A_i \in \mathcal{K}(\kappa) \div [A_1, \ldots, A_{i-1}]$, then $\kappa_{\div[A_1,\ldots,A_{i-1}]}(\bar{A}_i) > 0$, and hence $C(\mathcal{K}(\kappa) \div [A_1, \ldots, A_i]) = C(\mathcal{K}(\kappa) \div [A_1, \ldots, A_{i-1}]) \cup \{w \in \bar{A}_i | \kappa(w) \leq \kappa(w')$ for all $w' \in \bar{A}_i\} = C(\mathcal{K}(\kappa) \div [A_1, \ldots, A_{i-1}]) \cup C(\mathcal{K}(\kappa) \div A_i)$. Thus, $\mathcal{K}(\kappa) \div A_1 \cap \cdots \cap \mathcal{K}(\kappa) \div A_i \subseteq \mathcal{K}(\kappa) \div [A_1, \ldots, A_i]$. This inductively proves the reverse direction.

I take this to be a desirable theorem. It might have been difficult to motivate it as a definition of package contraction; but if it is a consequence of a plausible explication, this establishes mutual support for the explication and the theorem. In some sense, the theorem may also be disappointing. It says that package contraction is reducible to ordinary single contraction, after all, and is not an independent general issue.

I am not sure whether I am thereby contradicting Fuhrmann and Hansson (1994). They have doubts about (1) (see there p 62) and hence about the ensuing assertions. However, the doubts are raised only on their weaker axiomatic basis intended to leave room for denying (1)–(4). Hansson (1999) no longer comments on the properties (1)–(4). Thus, the only disagreement we may have is that I cannot share the doubts about (1), find my explication in definition 8 utterly plausible, and do not see any need, hence, to retreat to a weaker axiomatic characterization.

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Chapter 25
Statistical Inference Without Frequentist Justifications

Jan Sprenger

25.1 Frequentist Statistics and Frequentist Justifications

In modern science, inductive inference often amounts to statistical inference. Statistical techniques have steadily conquered terrain over the last decades and extended their scope of application to more and more disciplines. Explanations and predictions, in high-level as well as in low-level sciences, are nowadays fueled by statistical models. However, this development did not occur because scientists believe the underlying systems to be irreducibly stochastic. This might sometimes be the case, but certainly not in general. Rather, even traditionally "deterministic" sciences (such as several branches of physics, psychology and economics) use statistics to model noise and imperfect measurement and to express their uncertainty about the nature of the data-generating process. A wide spectrum of techniques can be used to draw valid conclusions from data: Hypothesis tests help scientists to see which of two competing hypotheses is better supported. Confidence intervals narrow down the set of values of an unknown model parameter which is compatible with the observations. And so on.

The classical methodology to answering these questions is *frequentist inference* (cf. Cox 2006). For reasons that will soon become obvious, I believe the term "frequentism" to be a misnomer. Rather, as pointed out by Mayo (1996), that school of statistical inference is characterized by a focus on the probability of making an error in the inference to a certain hypothesis or in setting up a confidence interval – hence, the name *error statistics*. A statistical procedure is good if and only if the two probabilities of committing an error – accepting a hypothesis when it is false, rejecting it when it is true – are low. For instance, assume that you want to test whether in a culture of 10,000 cells, less than 5% have been infected with a certain virus. That is your working hypothesis. To perform the test, you draw a sample of 100 cells. Then you formulate a decision rule whether or not to accept that hypothesis, dependent on

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M. Suárez et al. (eds.), *EPSA Epistemology and Methodology of Science: Launch of the European Philosophy of Science Association*