Chapter 7
Absolute Infinity in Class Theory and in Theology

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How can I talk to you, I have no words
Virgin Prunes, *I am God*

Abstract In this article we investigate similarities between the role that ineffability of Absolute Infinity plays in class theory and in theology.

Keywords Absolute infinity · God · Reflection principles

7.1 Introduction

Zermelo held that there exist no collections beside sets. According to most interpretations—and I will go along with those here—he held that the mathematical universe forms a potentially infinite sequence of sets of a special kind, which he called 'normal domains'. Quantification over sets is then necessarily restricted: we cannot quantify over all sets.

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Cantor held that the set theoretic universe exists as a *completed* absolute infinity. The Burali-Forti paradox and Russell's paradox were initially interpreted as showing that Cantor's 'naive' set theory, as it is still sometimes called, is inconsistent. It was thought that Cantor had failed to recognise that the mathematical universe cannot itself constitute a set. Cantor himself protested that he never took the set theoretic universe as a whole to be a set. Nowadays, Cantor is rarely accused of having defended an outright inconsistent theory of sets. Nevertheless, according to the received view, Cantor's views about the set theoretic universe as whole are outdated, and ultimately philosophically untenable.[1]

Certainly there are, as we shall see, tensions in Cantor's view of the nature of the existence of the set theoretic universe. But a Cantorian viewpoint, when appropriately understood, is in some sense more powerful and fruitful than Zermelo's view of the set theoretic universe. This is manifested in the motivation of reflection principles in set theory. These are principles that state, roughly, that there are some sets in the universe that in certain ways resemble the universe as a whole (which is "too large" to be a set). The more the set theoretic universe resembles certain sets, the harder it becomes to distinguish it from them, i.e., the more ineffable it becomes. It is known that on Zermelo's conception of the set theoretic universe, at best only weak reflection principles can be motivated, which give rise to small large cardinal principles (Zermelo 1996). It is also known that a Cantorian conception of the set theoretic universe that countenances proper classes, can motivate somewhat stronger reflection principles (Bernays 1961; Tait 2005). We will see that a Cantorian viewpoint in fact motivates much stronger reflection principles, from which much stronger large cardinal axioms can be derived.

In western theology concepts of ineffability of the Absolutely Infinite have always played an important role. We will see that in some theological traditions, this ineffability is, like in class theory, understood in terms of indistinguishability of the Absolute (God) from certain entities that are in fact distinct from the Absolute. So there is a similarity between the role that the ineffability of Absolute Infinity plays in certain theological views and in class theory. The principal aim of the present article is to describe and explore this similarity.

7.2 Zermelo

Zermelo was the first to hold that, *Urelemente* aside, the mathematical universe consists only of sets. Through the work of Zermelo, Fraenkel and von Neumann, it became established in the 1920s that sets are governed by the laws of *ZFC*. This has become the most prevalent form of set theoretic platonism: there are only sets, and they obey the principles of *ZFC*.

[1] For one expression of this view, see Jané (1995).

The question then arises how the sets are related to the mathematical universe. Zermelo's viewpoint can arguably be canvassed as follows (Zermelo 1996, pp. 1231–1233). When we are engaged in set theory, our quantifiers always range over a domain of discourse D, which Zermelo calls a 'normal domain'. The entities over which our set theoretic quantifiers range are sets: they are governed by the principles of standard set theory (*ZFC*). Our domain of discourse D itself is also a collection. Since there are no collections other than sets (and *Urelemente*, for Zermelo, but we disregard them here), our domain of discourse must also be governed by the principles of *ZFC*. But, on pain of contradiction, D can then not be included as an element in our domain of discourse. Nonetheless, we can expand our domain of discourse so that it includes D as an element. The expanded domain of discourse D' can even be taken to be such that it also satisfies the principles of *ZFC*. But the expanded domain D' will again be a set. So the previous considerations apply to D' also: it cannot contain itself as an element, even though we can expand it further so as to remedy this defect. In sum, even though the domain of discourse can always be expanded, it never comprises all sets. The upshot is that for Zermelo, the mathematical universe is a potential infinite sequence of (actually infinite) domains of discourse that satisfy the principles of *ZFC*:

> What appears in one model as an 'ultrafinite non- or super-set' is in the succeeding model already a perfectly good, valid 'set' with a cardinal number and ordinal type, and it is itself a foundation-stone for the construction of a new domain. To the unlimited series of Cantor's ordinal numbers there corresponds a likewise unlimited double series[2] of essentially different set-theoretic models in each of which the whole classical theory is expressed. The two opposite tendencies of the thinking spirit, the idea of creative *advancement* and that of collective completion [*Abschluss*] [...] are symbolically represented and reconciled in the transfinite number series based on well-ordering. This series in its unrestricted progress reaches no true completion; but it does possess relative stopping points, namely those 'limit numbers' which separate the higher from the lower models. (Zermelo 1996, p. 1233)

There are basic structural insights about the set theoretic universe that escaped Cantor. For instance, Zermelo in his later years viewed the set theoretic universe as structured into a layered hierarchy of initial segments V_α (with α ranging over the ordinals) that are sets. Zermelo even saw that there might be segments V_α that in a strong sense make all the axioms of *ZFC* true, as the quotation shows. Cantor did not see that far.

Zermelo's picture does raise some difficult questions. For one thing, it is not clear in which dimension the mathematical universe is supposed to vary. The notion of *creative advancement* suggests some form of progression or growth but it is not easy to see what the literal content of this metaphor really is. For another, there is the question how Zermelo can get his view across to us. (*What* is it that we cannot quantify over?) This does not imply that Zermelo's thesis about the essential restrictedness of quantification is false; however, it does seem difficult to see how this thesis can be communicated. I shall not pursue these worries here, but merely note that this can be taken as a reason for preferring Cantor's view over Zermelo's.

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[2]Zermelo here means a series of initial segments V_α of the set theoretic universe V, for α ranging over the strongly inaccessible ordinals, and the membership relation restricted V_α.

7.3 Cantor on the Set Theoretic Universe

Cantor's theory of the nature of the set theoretic universe as a whole is not easy to summarise. His views seem to have undergone a transformation around 1895. I first discuss his earlier views, and then turn to his later views.

7.3.1 The Absolutely Infinite

Cantor's basic convictions preclude Zermelo's potential infinity of (completed) normal domains ever to be the final word about the nature of the set theoretic universe. The set theoretic universe could not, in Cantor's view, form a potential infinity of actual infinities because of what Hallett calls Cantor's *domain principle* (Hallett 1984, pp. 7–8), which says that every potentially infinite variable quantity presupposes an underlying fixed and completed domain over which the potentially infinite entity varies:

> There is no doubt that we cannot do without variable quantities in the sense of the potential infinite; and from this the necessity of the actual infinite can also be proven, as follows: In order for there to be a variable quantity in some mathematical inquiry, the 'domain' of its variability must strictly speaking be known beforehand through a definition. However, this domain cannot itself be something variable, since otherwise each fixed support for the inquiry would collapse. Thus, this 'domain' is a definite, actually infinite set of values. Thus, each potential infinite, if it is rigorously applicable mathematically, presupposes an actual infinite. (*Mitteilungen zur Lehre vom Transfiniten VII* (1887): (Cantor 1932, pp. 410–411), my translation)[3]

In particular, this means that even every absolute infinity of transfinite sets that potentially exists presupposes an actual, completed absolutely infinite domain as its range of variation.

Admittedly Cantor was in his writings not very explicit about what he did take the set theoretic universe as a whole to be. One problem is that it is not in every instance clear whether he has a theological or a mathematical conception of absolute infinity in mind. Indeed, he argues that it is the task not of mathematics but of 'speculative theology' to investigate what can be humanly known about the absolutely infinite (Cantor 1932, p. 378).[4] The following passage, for example, leans heavily to the theological side:

> I have never assumed a "Genus Supremum" of the acual infinite. Quite on the contrary I have proved that there can be no such "Genus Supremum" of the actual infinite. What

[3]In this quotation, Cantor speaks of the necessity of 'knowing' the domain of variation through a 'definition'. Surely Cantor is merely sloppy here, and we should discount the epistemological overtones. Another slip can be detected in Cantor's use of the word 'set' in this quotation: Cantor means the argument to be applicable not just to sets but also to absolute infinities. For a discussion of Cantor's sometimes sloppy uses of the term 'set', see Jané (2010), footnote 60.

[4]The connection between Cantor's conception of the mathematical absolutely infinite and his conception of God is explored in van der Veen and Horsten (2014).

> lies beyond all that is finite and transfinite is not a "Genus"; it is the unique, completely individual unity, in which everything is, which comprises everything, the 'Absolute', for human intelligence unfathomable, also that not subject to mathematics, unmeasurable, the "ens simplicissimum", the "Actus purissimus", which is by many called "God". (Letter to Grace Chisholm-Young (1908): (Cantor 1991, p. 454), my translation).

All this is related to the fact that in an Augustinian vein, Cantor takes all the sets to exist as ideas in the mind of God[5]:

> The transfinite is capable of manifold formations, specifications, and individuations. In particular, there are transfinite cardinal numbers and transfinite ordinal types which, just as much as the finite numbers and forms, possess a definite mathematical uniformity, discoverable by men. All these particular modes of the transfinite have existed from eternity as ideas in the Divine intellect. (Letter to Jeiler (1895): (Tapp 2005, p. 427), my translation)

Even though for this reason mathematical entities (sets and proper classes) are for Cantor not distinct from God, it is clear that he at times has a mathematical conception of the Absolutely Infinite in mind:

> The transfinite, with its wealth of arrangements and forms, points with necessity to an absolute, to the 'true infinite', whose magnitude is not subject to any increase or reduction, and for this reason it must be quantitatively conceived as an absolute maximum. (*Mitteilungen zur Lehre vom Transfiniten V* (1887): (Cantor 1932, p. 405), my translation)

This is the notion of absolutely infinite that I shall concentrate on in this article. I shall from now on disregard what Cantor takes to be the theological aspects of the mathematical absolutely infinite; I shall instead concentrate on Cantor's conception of the 'quantitatively absolute maximum', which is the set theoretic universe as a whole. From the passages discussed above, I conclude that he attributes to it the following properties. It is a fully determinate, fully actual ('completed'), inaugmentable totality. It is composed of objects (sets) that are of a mental nature ('ideas'). And unlike the sets in the mathematical universe, the universe as a whole cannot be uniquely characterised.

7.3.2 *Inconsistent Multiplicities*

From around the time when Burali-Forti published his 'paradox' (Burali-Forti 1897), one finds a subtle change of terminology in Cantor's writings. Whereas before, Cantor used the expression 'the Absolutely Infinite' for characterising the set theoretic universe, he now categorises the set theoretic universe and other proper classes (such as the class of all ordinals) as *inconsistent multiplicities*:

> If we assume the concept of a determinate multiplicity (of a system, of a realm ['Inbegriff'] of things), then it has proved to be necessary to distinguish two kinds of multiplicity (I always mean determinate multiplicities).

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[5] For Cantor's most detailed account of the set theoretic universe in God's mind, see Tapp (2005), pp. 414–417. See also *Mitteilungen zur Lehre vom Transfiniten V*, footnote 3 (Cantor 1932, pp. 401–403).

> A multiplicity can be of such nature, that the assumption of the 'togetherness' ['Zusammenseins'] of its elements leads to a contradiction, so that it is impossible to conceive the multiplicity as a unity, as a 'finished thing'. I call such multiplicities absolutely infinite or inconsistent multiplicities. (Letter to Dedekind (1899), (Cantor 1932, p. 443), my translation)

Jané has argued that passages such as these indicate that Cantor no longer believed that the set theoretic universe forms a completed infinity (Jané 1995, Sects. 6 and 7). The strongest evidence for this thesis is perhaps the following quote from a letter from 1899 from Cantor to Hilbert:

> I am now used to call 'consistent' what before I referred to as 'completed', but I do not know if this terminology deserves to be maintained. (Letter to Hilbert (1899): (Cantor 1991, p. 399))

Jané speculated in Jané (1995) that instead of conceiving of the set theoretic universe as a completed whole, Cantor tacitly moved to a conception of the set theoretic universe as an irreducibly potential entity, whereby he arrived at a prefiguration of Zermelo's conception of the mathematical universe. This means that he must have by that time tacitly given up on the domain principle which says that every potential infinite has as its domain of variation an underlying completed infinite.

In his more recent (Jané 2010), Jané no longer claims that Cantor actually gave up the thesis of the existence of the mathematical universe as a completed infinity. But Jané rightly stresses that there remains a tension between Cantor's earlier commitments and Cantor's later terminology of inconsistent multiplicities:

> I submit that, owing to Cantor's allegiance to a changeless mathematical universe, Cantor's explanations [of the concept of inconsistent multiplicity] are indeed unconvincing. For how can the elements of a multiplicity fail to coexist if they all inhabit the same universe? (Jané 2010, p. 223)

And he thinks that the best way for Cantor to resolve this tension would be to embrace Zermelo's conception of the set theoretic universe as essentially open-ended.

Not everyone agrees with Jané's interpretation. It is true that Cantor's choice of words in the letter to Hilbert indicates that he no longer believed that the set theoretic universe can be mathematically understood as a whole. But the passages do not show that Cantor no longer believed that the set theoretic universe does not form an inaugmentable totality that forms the domain of our mathematical discourse past, present, and future. In Hauser's words:

> [B]y 'existing together' Cantor evidently means 'existing together as elements of a "finished" set'. Thus, what he is saying is merely that the totality of all transfinite numbers (or all alephs) does not constitute a set and therefore cannot be an element of some other set. But he is not denying that the transfinite numbers coexist in some other form, namely as *apeiron*, which is mathematically indeterminate, meaning that one cannot assign a cardinal or ordinal number to the totality of all numbers (Hauser 2013, Sect. 3).

The content of the notion 'apeiron' is notoriously unclear.[6] So this does not really help much in the clarification of the nature of the set theoretic universe. In other words,

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[6]This notion goes back to Anaximander, and is variously translated as 'limitless','boundless', 'formless', 'the void'....

there is an unresolved interpretative debt at this point on the side of the defender of the Cantorian viewpoint. It seems that Jané is right that Cantor (or his defender) is facing a choice. Either she upholds Cantor's earlier view of the set theoretical universe and tries to make good philosophical sense of it, or she takes Cantor's characterisation of the mathematical universe as an inconsistent multiplicity as the final word, and tries to make sense of that. But both cannot be done at the same time.

What I propose to do is in the first instance to ignore Cantor's description of the set theoretic universe as an inconsistent multiplicity. In the following sections, I shall adopt Cantor's characterisation of the set theoretic universe as a completed whole, and discuss how it can be used to motivate what are called 'top down' reflection principles. Then I shall discuss a stronger reflection principle. We shall see that to make sense of this stronger reflection principle, elements both of Cantor's earlier views and elements of Cantor's later views on the nature of the set theoretic universe can be used.

7.4 Reflection

According to a time-honoured and influential view in the Judeo-Christian theological tradition, God is fundamentally ineffable. Cantor was well aware of this tradition and he extended it to mathematical absolutely infinites. After Cantor's time, in modern set theory, this view has been given *positive* expressions, which somewhat surprisingly have mathematical strength. These statements are known as *reflection principles*.

7.4.1 The Very Idea

The starting point of set theoretic reflection is the early Cantorian view that the mathematical absolutely infinite is unknowable:

> The Absolute can only be acknowledged, but never known, not even approximately known. (*Grundlagen einer allgemeinen Mannigfaltigkeitslehre* (1883), endnote to Sect. 4: (Cantor 1932, p. 205))

There are obvious connections with central themes in theology, especially with the medieval doctrine that only negative knowledge is possible of God (apophatic theology). As it stands, it is indeed a negative statement. However it can be given a positive interpretation as follows. Let us provisionally identify the mathematical absolutely infinite with the set theoretic universe as a whole (V). The universe V is unknowable in the sense that we cannot single it out or pin it down by means of any of our assertions: no true assertion about V can be made that excludes unintended interpretations that make the assertion true. In particular—and this is stronger than the previous sentence—no assertion that we make about V can ensure that we are talking about the mathematical universe rather than an object *in* this universe. So if we do make a true assertion ϕ about V, then there exist sets s such that ϕ is also true when it is interpreted in s.

In the late 1890s the Burali-Forti theorem made it abundantly clear that V is not the only actual whole that is absolutely infinite: the ordinals, for instance, form an absolutely infinite sequence. So in light of this we must say that the mathematical absolutely infinite comprises, in addition to the mathematical universe as a whole all other proper classes.[7] But in fact, the above argument should hold true for any proper class. Proper classes can then be said to be unknowable in the sense that no assertion in the language of sets can be true of *only* for some proper classes. So if we do make a true assertion concerning a proper class, then there exists sets about which this assertion is already true. If we truly describe mathematical absolute infinities, then there are set proxies for the absolute infinities such that our description can also truly be taken to range over the proxies.

Cantor did not explicitly articulate this line of argument. Yet he was probably the first one to make use of reflection as a principle motivating the existence of sets. He argues that the finite ordinals form a set because they can be captured by a definite condition:

> Whereas, hitherto, the infinity of the first number-class (I) alone has served as such a symbol [of the Absolute], for me, precisely because I regarded that infinity as a tangible or comprehensible idea, it appeared as an utterly vanishing nothing in comparison with the absolutely infinite sequence of numbers. (*Grundlagen einer allgemeinen Mannigfaltigkeitslehre* (1883), endnote to Sect. 4: (Cantor 1932, p. 205))

This can be seen as an application of a reflection principle.[8] Being closed under the successor operation is a set theoretic property of a mathematical absolute infinity (the ordinals). Reflection then allows us to infer that there must be a *set* that is closed under the successor operation, and hence that there must be a minimal such. This is the set ω. Anachronistically, one is tempted to say that Cantor is appealing to something like Montague-Levy reflection, which is a first-order reflection property that is provable in *ZFC*.[9]

7.4.2 Set Reflection

On the face of it, Zermelo's viewpoint uses a form of set theoretic reflection: every admissible domain of discourse in set theory is a 'normal domain', and this can by a reflective movement be seen to be a set. We cannot quantify over or in any way make use of proper classes, for, in his view, no such things exist. The set theoretic universe as a whole is not something we can talk about, according to Zermelo, for it

[7] Cantor's 1899 argument that the ordinals form an inconsistent totality is critically discussed in Jané (1995), pp. 395–396.

[8] Admittedly this passage is sufficiently vague as to be open to multiple interpretations. The view that this passage should be seen as an application of reflection is defended in Hallett (1984), pp. 117–118.

[9] The Montague-Levy reflection principle is discussed in Drake (1991), Chap. 3, Sect. 6.

never exists as a completed realm. So, literally speaking, Zermelo cannot, according to his own view, truly say that "*the* set theoretic universe is so rich that it contains many normal domains".

The best Zermelo can do is simply to postulate that above every ordinal, there is an ordinal which is the 'boundary number' of a normal domain. In modern terms, this is expressed as an axiom that postulates unboundedly many strongly inaccessible cardinals:

Axiom 1 $\forall\alpha\exists\beta : \beta > \alpha \wedge$ "β is a strongly inaccessible cardinal" .

This *seems* to say exactly what is required. It says that a fundamental property of the set theoretic universe, namely making the axioms of standard second-order set theory (ZFC^2) true,[10] is reflected in arbitrarily large set-sized domains. But closer inspection reveals that this cannot exactly be the case: there must be ordinal numbers that fall outside the quantifiers in this axiom. By Zermelo's own lights, the quantifiers in Axiom 1 must range over a domain of discourse that forms a set in a wider domain of discourse. There will be ordinals in this wider domain of discourse that do not belong to the 'earlier' domain of discourse.

Nonetheless, Axiom 1 and its relatives have some proof theoretic strength. They postulate the existence of 'small large cardinals' to which *ZFC* is not committed (Drake 1991, Chap. 4). That these large cardinals are still relatively small is witnessed by the fact that it is consistent for them to exist in Gödel's constructible universe L.

7.4.3 Class Reflection

Stronger reflection principles can be formulated if we take Cantor's idea of absolutely infinite multiplicities seriously. However, to study these reflection principles in a precise setting, logical laws governing them have to be formulated. The language that is assumed is the language of second-order (or two-sorted, if you will) set theory, where the membership symbol is expressing the only fundamental non-logical relation, and where we have two types of variables: the first-order variables range over sets ($x, y, \ldots$) and the second-order variables range over (proper and improper) classes ($X, Y, \ldots$). we shall from now on take the sets and classes to be governed by the principles of Von Neumann–Bernays–Gödel (NBG) class theory (and worry about the justification for this later).[11] Indeed, von Neumann's class theory, the precursor to Bernays' formulation of NBG, can be seen as a formalisation of Cantor's viewpoint (but not as a conceptual clarification).[12]

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[10]Ranks V_α, for α strongly inaccessible, are models of ZFC^2.

[11]NBG differs from full ZFC^2 in that the second-order comprehension scheme is restricted to formulae that do not contain bound occurrences of second-order quantifiers.

[12]See von Neumann (1967).

If we take the point of view of Cantor's early theory of the mathematical universe, and take the point that there are more absolutely infinite collections than V alone, then we can express the reflection idea as follows:

Axiom 2 $\forall X : \Phi(X) \rightarrow \exists \alpha : \Phi^{V_\alpha}(X \cap V_\alpha)$,

where Φ^{V_α} is obtained by relativising all first- and second-order quantifiers to V_α and its power set, respectively, and where α does not occur free in Φ.

Zermelo's reflection principle (Axiom 1) only expresses that certain true class theoretical statements are reflected downwards (the axioms of ZFC^2). Axiom 2 states that *every* true (second order parametrised) class theoretic statement is reflected down to some set sized domain. Axiom 2 is stronger than Axiom 1: it implies large cardinal principles that postulate indescribable cardinals.[13]

Of course it is then natural to formulate reflection principles of orders higher than two in an analogous manner. However already the full third-order class reflection principle is inconsistent, at least for formulae that involve general parameters (Reinhardt 1974), (Koellner 2009, Sect. 3).[14] Third-order reflection restricted to a certain class of "positive" formulae is consistent and stronger than second-order class reflection (Tait 2005), but does not prove the existence of measurable cardinals or any other cardinals that are incompatible with $V = L$. Fourth-order reflection is inconsistent even when restricted to "positive" formulae (Koellner 2009, Sect. 5).

In sum, the situation is this. From Zermelo's conception of the set theoretic universe as a potential infinity of sets, the region of small large cardinals in the neighbourhood of inaccessible cardinals can be motivated. Due to its recognition of proper classes alongside of sets, the Cantorian point of view can be said to lead to the above stronger reflection principles of class reflection. However even those principles do not take us beyond the small large cardinal principles consistent with V being Gödel's constructible universe L. Indeed, the (tentative) conclusion of Koellner (2009) is that class theoretic reflection principles are either weak (in terms of large cardinal strength) or inconsistent.

7.5 Global Reflection

Gödel thought that *all* sound large cardinal principles can be reduced to reflection principles:

> All the principles for setting up the axioms of set theory should be reducible to Ackermann's principle: The Absolute is unknowable. The strength of this principle increases as we get stronger and stronger systems of set theory. The other principles are only heuristic principles. Hence, the central principle is the reflection principle, which presumably will be understood better as our experience increases. Meanwhile, it helps to separate out more specific principles

[13] Axiom 2 and its relatives were discussed in Bernays (1961). For a discussion of indescribable cardinals, see Drake (1991), Chap. 9.

[14] Parameter free sentences of higher orders are unproblematic.

> which either give some additional information or are not yet seen clearly to be derivable from the reflection principle as we understand it now (Wang 1996, 8.7.9).

This sentiment goes against the conclusions that Koellner reached, and is often regarded as implausible, because the familiar reflection principles are compatible with the principle that $V = L$. Nonetheless, we shall now argue that from a Cantorian point of view there may be more to Gödel's conjecture than is commonly thought.

Gödel himself was an adherent of Cantor's actualist viewpoint regarding the set theoretic universe rather than of Zermelo's potentialist viewpoint:

> To say that the universe of all sets is an unfinished totality does not mean objective undeterminateness, but merely a subjective inability to finish it. (Gödel, as reported in Wang (1996), 8.3.4)

We have seen that the set theoretic universe as a whole and all classes of sets are recognised by Cantor to (actually) exist: let us call this structure $\langle V, \in, C \rangle$, where C contains all of Cantor's absolutely infinite collections. Then the reflection idea tells us that we cannot single this structure out by means of any of our assertions. Positively put, any assertions that hold in $\langle V, \in, C \rangle$ must also hold in some set-size structure.[15]

There are various possible ways of trying to making this more precise. I shall not try to give a catalogue of the pro's and contra's of various options. Rather, I shall concentrate on one way that seems especially powerful, natural, and fruitful. As was mentioned earlier, it is to be assumed that $\langle V, \in, C \rangle$ makes at least the principles of NBG true. Against this background, Welch proposed the following principle (Welch 2012):

Axiom 3 There is an ordinal κ and a nontrivial elementary embedding

$$j : \langle V_\kappa, \in, V_{\kappa+1} \rangle \longrightarrow \langle V, \in, C \rangle$$

with critical point κ (i.e., $j(\kappa) > \kappa$ whereas below κ, j is the identity transformation).

This principle is called the *Global Reflection Principle* (GRP).[16] What the embedding function does is to act as the identity function on all elements of V_κ but to send the elements of $V_{\kappa+1}$ to elements of C: $j(\kappa) = On$, $j(V_\kappa) = V$, $j(Card \cap \kappa) = Card, \ldots$ So Axiom 3 says that the set theoretic universe (with all its proper classes) is reflected in a particular way to a set-size initial segment of the universe.

The level of elementarity that is insisted upon can be varied. For the most part of this article we will require only Σ^0_1-elementarity where formulae are allowed to have class variables $X, Y, X, \ldots$ but are restricted to have only existential quantifiers which range over sets alone: we denote the resulting global reflection principle as $GRP_{\Sigma^0_1}$. But we could also insist on Σ^0_∞-elementarity or even Σ^1_∞-elementarity

[15] We will see later (Sect. 7.6.3) that the expression "any assertions" in this statement may need to be qualified.

[16] A philosophical defence of GRP is given in Horsten and Welch (forthcoming).

(denoted as $GRP_{\Sigma^0_\infty}$, $GRP_{\Sigma^1_\infty}$, respectively). Often, however, I will leave the level of elementarity required by the principle unspecified and simply speak of *GRP*.

The principle *GRP* says that the universe with its parts is, to a certain degree, indistinguishable from at least one of its initial parts V_κ and *its* parts. It says that the whole set theoretic universe with all its proper classes is mirrored in a set-sized initial segment $\langle V_\kappa, \in, V_{\kappa+1}\rangle$, where the first-order quantifiers range over V_κ, and where the reflection of a proper class C is obtained by 'cutting it off' at level V_κ.

GRP expresses the idea of reflection in a more powerful way than Axiom 2. Axiom 2 just says that each (second-order) statement is reflected from the set theoretic universe to some V_κ (where possibly different second-order statements are reflected in different V_κ's): therefore it does not entail that the universe as a totality particularly resembles any one *single* set-like initial segment. However *GRP* postulates that the whole universe $\langle V, \in, C\rangle$ is indistinguishable from an initial 'cut' $\langle V_\kappa, \in, V_{\kappa+1}\rangle$ in a very specific way, namely in a way such that no large 'set' and no proper class can be distinguished from a proper subset of itself (its intersection with V_κ and with $V_{\kappa+1}$, respectively).

Even the weaker versions of *GRP* have strong large cardinal consequences. They entail the existence of sets that are incompatible with $V = L$, such as measurable cardinals, Woodin cardinals,...[17]

Thereby *GRP* is a more robustly ontological form of reflection than Axiom 2. In this respect, there is a striking connection with theological ideas that have a long history, as the following passage shows (Odo Reginaldus, quoted in Côté (2002), p. 78, my translation):

> How can the finite attain [knowledge of] the Infinite? On this question some said that God will show Himself to us in a mediated way, and that he will show Himself to us not in His essence, but in created beings. This view is receding from the aula...[18]

Of this passage, van Atten remarks (van Atten 2009, footnote 84, p. 22):

> From here it is only a small step to: "Suppose creature *A* has a perception of God. Then God is capable of making a creature *B* such that *A*'s perception cannot distinguish between God and *B*."

Indeed, I conjecture that the "view that is receding from the aula" to which Reginaldus is referring traces back to Philo of Alexandria, who writes in his *On Dreams*[19]:

> Thus in another place, when he had inquired whether He that is has a proper name, he came to know full well that He has no proper name, [the reference is to Exodus 6:3] and that whatever name anyone may use for Him he will use by licence of language; for it is not in the nature of Him that is to be spoken of, but simply to be. Testimony to this is afforded also by the divine response made to Moses' question whether He has a name, even "I am He that is (Exodus 3:14)." It is given in order that, since there are not in God things that man can

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[17]The large cardinal strength of versions of *GRP* is discussed in Horsten and Welch (forthcoming) and in Welch (2012).

[18]"Quomodo potest finitum attingere ad infinitum? Propter hoc dixerunt alii quod deus contemperatum se exhibebit nobis, et quod ostendet se nobis non in sua essentia, sed in creatura".

[19]As quoted in Segal (1977), p. 163.

comprehend, man may recognise His substance. To the souls indeed which are incorporeal and occupied in His worship it is likely that He should reveal himself as He is, conversing with them as friend with friends; but to souls which are still in the body, giving Himself the likeness of angels, not altering His own nature, for He is unchangeable, but conveying to those which receive the impression of His presence a semblance in a different form, such that they take the image to be not a copy, but that original form itself.

Although we have seen that Cantor was deeply familiar with the idea of God as ineffable, there is no textual evidence to suggest that he was familiar with theological literature in which the uncharacterisability of God is transformed into a *positive* principle, as was done in the passages above. Yet we have seen that Cantor at least once more or less explicitly made use of a mathematical reflection principle. But then it was done only in a fairly minimal way, namely, to argue for the existence of ω as a set. *GRP* is clearly a *much* stronger reflection principle than the one that Cantor implicitly appealed to in the quoted passage (Montague-Levy). But it is the class-theoretic counterpart of the theological thesis that is defended by Philo of Alexandria. Just as in the theological context, there are 'angels' such that every humanly describable property of God also applies to them, in the class theoretic context there are some sets such that every property of the universe also holds when relativised to them.

A key difference between the theological case and the class theoretic case is that we do not have a good theoretical understanding of the 'angels' in question, whereas we do have an excellent theory of sets. Also, in class theory, absolute infinities are reflected *in* the mathematical universe, whereas in the theological case, God is reflected in beings outside Himself.[20]

7.6 Sets, Parts, and Pluralities

Now that the philosophical motivation behind, and the content of, *GRP* has been explained, we turn to the ontological assumptions of the framework in which it is formulated.

7.6.1 GRP as a Second-Order Principle

So far we have only expressed *GRP* in a semi-formal way—in a manner of speaking often adopted by set theorists. If we formally want to express *GRP*, then at first blush it seems that we need a language of third order: the function j that is postulated to exist pairs sets of V_κ with themselves and sets of $V_{\kappa+1}$ with proper classes. Yet on the Cantorian perspective that we have adopted so far, only sets and collections of sets (proper classes) have been countenanced. But of course the mapping j that is

[20] I am indebted to an anonymous referee for these points.

postulated by *GRP* can in fact be *coded* as a second-order object: as a proper class K consisting of ordered pairs such that its first element a is in the domain of j (namely: $V_{\kappa+1}$) and the second element $j(a)$ is an element of $V_\kappa \bigcup \mathcal{C}$.

We also need a satisfaction predicate to express the elementarity of the embedding. *GRP* deploys two notions of truth: truth in the structure $\langle V_\kappa, V_{\kappa+1}, \in\rangle$, and truth in $\langle V, \mathcal{C}, \in\rangle$. Truth in $\langle V_\kappa, V_{\kappa+1}, \in\rangle$ can of course be defined in the language $\mathcal{L}^2_\in$. Σ^0_1-truth in $\langle V, \mathcal{C}, \in\rangle$ is also definable in $\mathcal{L}^2_\in$. But Σ^1_∞-truth is not (by Tarski's theorem). For the expression of the version of *GRP* that postulates Σ^1_∞-elementarity of j, it suffices to have a primitive Tarskian compositional satisfaction predicate T to $\mathcal{L}^2_\in$ and insist that the compositional truth axioms hold for $\mathcal{L}^2_\in$. This suffices to express what it means for a statement of $\mathcal{L}^2_\in$ to be true and to prove basic properties of truth. So, in sum, the fact that *GRP* postulates the elementarity of the embedding j even if we take a strong version of GRP that is Σ^1_∞ preserving, does not necessitate us to go up to third order.

7.6.2 Parts of V

As mentioned earlier, Cantor's distinction between sets and Absolute Infinities is a prefiguration of the distinction between sets and proper classes, which was articulated explicitly by von Neumann. The difference with Cantor's theory is that von Neumann did take classes as well as sets to be governed by mathematical laws. It is just that classes are objects *sui generis*: they obey different laws. Proper classes are objects that have elements, but they are not themselves elements. So, in particular, there is no analogue of the power set axiom for proper classes.

However it remained an open question how talk of proper classes ought to be interpreted. In particular, if proper classes are taken to be super-sets in some sense, then it is somewhat mysterious why they can have elements but not be elements. In Maddy's words (Maddy 1983, p. 122):

> The problem is that when proper classes are combinatorially determined just as sets are, it becomes very difficult to say why this layer of proper classes a top V is not just another stage of sets we forgot to include. It looks like just another rank; saying it is not seems arbitrary. The only difference we can point to is that the proper classes are banned from set membership, but so is the κth rank banned from membership in sets of rank less than κ.

And then why is there no singleton, for instance, that contains the class of the ordinals as its sole element? An alternative would be to say that proper classes can be collected into new wholes, but that these could (for obvious reasons) not themselves be proper classes. They would be again a *sui generis* kind of objects: super-classes. But in this way we embark on a hierarchical road that few find worth traveling. On this picture, classes, super-classes, et cetera, look too much like sets. We seem to be replicating the cumulative hierarchy of sets whilst incurring the cost of introducing a host of different kinds of set-like objects.

I propose instead to adopt a *mereological* interpretation of proper classes.[21] On this view, the mathematical universe is a mereological whole, and classes, proper as well as improper, are parts of the mathematical universe. We can identify those parts of V that are also parts of a set, i.e., that are set sized, with sets. The threat of a hierarchy of super- and hyper-wholes is not looming here. The fusion of the parts of a whole does not create a super-whole, but just the whole itself. So there is no mereological analogue of the creative force of the power set axiom.[22]

Such a mereological interpretation of *classes* is similar to David Lewis' interpretation of *sets* (Lewis 1991, 1993). Lewis takes sets to be generated by the singleton function and unrestricted mereological fusion. So sets have subsets as their mereological parts. Similarly, in the interpretation of classes that is proposed here, every class is a fusion of some singletons, and classes have sub-classes (and not their elements) as their proper parts. Also sets will have sub-sets as their parts; but in contrast to proper classes, they *are* also elements (of sets and of classes).

The difference between the proposed interpretation of the range of the second-order quantifiers and Lewis' theory of classes is that on the proposed interpretation set theory is taken as given. Lewis regards the relation between an entity and its singleton as thoroughly "mysterious" (Lewis 1991, Sect. 2.1). Reluctantly, he takes it to be a structural relation (Lewis 1991, Sect. 2.6). Derivatively, there is, in Lewis' approach, something mysterious about all sets. This is not the view that is taken here. I assume set theory from the outset, and do not commit myself to any specific interpretation (reductive or non-reductive) of the membership relation. Given the singleton-relation that is part of set theory, the elementhood relation for classes can be explained in a straightforward way. Explaining the element-relation for sets is outside the scope of this article.

The mereological interpretation of classes satisfies the two desiderata that according to Maddy an interpretation of class theory has to satisfy simultaneously (Maddy 1983, p. 123)[23]:

1. Classes should be real, well-defined entities;
2. Classes should be significantly different from sets.

The first desideratum is satisfied because classes are just as real and well-defined as sets. The second desideratum is satisfied because the laws of parthood are significantly different from the laws governing sets.

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[21] See also Horsten and Welch (forthcoming).

[22] Even the Augustinian idea that sets are ideas in God's mind is compatible with this view. Within such a framework, the mereological conception of classes would result in conceiving of classes (proper and improper) as *parts* of God's mind.

[23] It seems to me that Maddy's own view of classes does not completely satisfy the first desideratum. The reason is that she takes the class membership relation to be governed by partial logic. According to her theory, there is in many cases no fact of the matter whether a given class is an element of another given class.

7.6.3 Mathematical and Mereological Reflection

Σ^0_∞ statements can be classified as as *mathematical* statements because they only quantify over sets. Σ^1_∞ statements can be classified as *mereological* (or in Cantorian vein one might say *theological*) statements because they quantify over proper classes, which on the proposed interpretation are regarded as extra- or supra-mathematical objects. In other words, we might call $GRP_{\Sigma^0_\infty}$ mathematical global reflection, whereas $GRP_{\Sigma^1_1}$ must already be regarded as mereological global reflection.

As mentioned before, already $GRP_{\Sigma^0_1}$ gives us strong large cardinal consequences. But one can go further than this and insist on mereological global reflection ($GRP_{\Sigma^1_1}$, for instance). In fact, even $GRP_{\Sigma^1_\infty}$ does not express the reflection idea in its strongest form. Recall that the guiding idea was that the set theoretic universe is *absolutely indistinguishable* from some set-like initial segment of V. *GRP* requires that the embedding j is elementary with respect to the second-order language of set theory *without the satisfaction predicate*. If we have a satisfaction predicate in our arsenal, we might require even stronger elementarity, viz. with respect to the second-order language *including the primitive satisfaction predicate*. Since this same satisfaction predicate is used to express the elementarity of j, it will have to be a non-Tarskian, type-free satisfaction predicate. For instance, one can contemplate using a truth predicate that is governed by axioms in the spirit of the Kripke–Feferman theory (Feferman 1991).

7.6.4 Names of God

In its most extreme form, negative theology states that no property can be truly predicated of God. In a more positive vein, one might say that everything that we can truly say about God, also holds for some being that is less exalted than God. (Note, once again, that this is not equivalent to the thesis in the first sentence of this paragraph.) Both these theories raise a difficult question, which was perhaps first articulated forcefully by Dionysius the Areopagite. If everything we truly say about God is also true about some angel(s), or if nothing we say about God is true at all, then how can we name God in the first place? What is it that makes our uses of the word 'God' refer to God rather than to angels in the first place?[24] The mirror image of this challenge for *GRP* is just as troublesome. If *everything* we truly say about V and $\mathcal{C}$ is also true about some set V_κ and its subsets, then what makes it the case that when we are using 'V' and '$\mathcal{C}$' in this article, these terms refer to the set-theoretic universe and its classes, respectively, rather than to some set and its subsets? If we insist on articulating *GRP* as requiring full Σ^1_∞ elementarity, then we only have the primitive notion of satisfaction to single out $\langle V, \mathcal{C}, \in \rangle$. However, if we articulate *GRP* as insisting only on a form of mathematical elementarity (Σ^0_1 elementarity or Σ^0_∞ elementarity), then this worry is not pressing. Then we can say that mereological

[24] See the quotation of Dionysius in Sect. 7.5.

(or "theological") statements might allow us to distinguish V and C from every set together with its subsets.

There is a less philosophical reason for perhaps being hesitant to endorse mereological reflection. $GRP_{\Sigma^1_\infty}$ entails the axioms MK of Morse–Kelley class theory, i.e., ZFC^2 with its impredicative class comprehension axiom. MK holds at $(V_\kappa, V_{\kappa+1}, \in)$, and is then sent up by virtue of the Σ^1_∞ elementarity of j. So one might as well have started with the very "class-impredicative" theory MK as one's background theory.[25] But against this, one might say that on an actualist conception of V and its parts, for the same reasons that impredicative definitions of sets are unproblematic,[26] impredicative definitions of classes are unproblematic also.

7.7 In Closing

According to many, Cantor's early view of the mathematical universe as a whole is hopelessly entangled with his theological views (Tapp 2014). In contrast, his later view of the set theoretic universe and proper classes more generally as 'inconsistent multiplicities' is less so, and can be seen as a first step in the direction of a modern view of the set theoretic reality. It can then be seen as a prefiguration of a potentialist conception of the mathematical universe *à* la Zermelo (Jané 1995).

In this article I have argued that good non-theological sense can be made of Cantor's earlier view of the set theoretic universe. Sets are all the mathematical objects there are. All the sets together form, as the early Cantor said, a completed whole: the mathematical universe V. However V itself is not a *mathematical* object. Proper classes are parts of the universe. Every part of V is a completed whole. Every set is an element of V. The parthood relation corresponds to the subclass relation, which is a transitive relation. So parthood is not the same as membership, even for sets: not all sets are transitive. The language of sets and parts of V is the language of second-order set theory $\mathcal{L}^2_\in$. The first-order quantifiers range over all sets. The second-order quantifiers range over all the parts of V. So we are ontologically committed to the existence of sets, the universe of all sets, and all of its parts: no further ontological commitments are made. The sets certainly satisfy *ZFC*. The parts of V satisfy at least predicative second-order comprehension. And the class replacement axiom also holds. So we are licensed to postulate NBG class theory in the language $\mathcal{L}^2_\in$. If one takes a Gödelian stance towards impredicative definitions, then even impredicative second-order comprehension is acceptable. If that is so, then the axioms of Morse-Kelly class theory are motivated.

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[25] This argument does not go through if instead j is only Σ^0_1 elementary: there is then not enough elementarity to preserve the impredicative second order comprehension scheme upwards. Nonetheless, since MK holds at $(V_\kappa, V_{\kappa+1}, \in)$, accepting $GRP_{\Sigma^0_1}$ still commits one to believing that impredicative second-order logic is at least coherent.

[26] See Gödel (1984).

Not only is this interpretation of Cantor's earlier view perfectly coherent. It is also mathematically fruitful. It allows us to indirectly motivate strong principles of infinity (large cardinal axioms). Large cardinal principles play an important role in contemporary set theory. However whereas the axioms of *ZFC* seem to be fairly generally accepted to hold of the set theoretic universe, there is no general agreement that most of the large cardinal principles hold.

Gödel argued that mathematical axioms can be motivated in two ways: intrinsically, and extrinsically (Gödel 1990). Extrinsic support for an axiom derives from its consequences. Thus extrinsic motivations are success arguments; they are instances of *Inference to the Best Explanation*. Many believe that intrinsic justification for mathematical principles is more reliable than extrinsic justification. Indeed, many do not think that external motivation for a mathematical axiom can provide strong confirmation of its truth (Tait 2001, p. 96). So it is an important question to what extent large cardinal principles can be motivated intrinsically.

Mathematical reflection principles are intrinsically motivated. These arguments follow a pattern of reasoning that has its roots in the Judeo-Christian theological tradition. This argument starts from the negative premise of the transcendence of God: there is no defining condition in any human language that is satisfied by Him and by Him alone. From this it follows that if we can truly ascribe a property to God, this property must hold of some entity that is different from God as well. This conditional positive statement can justly be called a *first theological reflection principle*. This argument can be strengthened if we assume the stronger negative premise that not even an infinite body of humanly describable conditions characterise God uniquely. This means that there must be an entity that is different from God and that satisfies all properties that can be truly ascribed to God. This then is a *second theological reflection principle*. We have seen how it was clearly articulated by Philo of Alexandria. The first theological reflection principle is the exact analogue of Bernays' second-order reflection principle. The second reflection principle is the analogue of the Global Reflection Principle.

Cantor did not see this far. On the theological side, there is no evidence that he was aware of statements of the second theological reflection principle. On the class theoretic side, he did not have the resources to even articulate the Global Reflection Principle: the cumulative rank structure of the set theoretic universe had yet to be discovered. We have seen that Cantor himself did (somewhat implicitly) appeal to a reflection principle on one occasion. But what he appealed to was a first-order reflection principle (Montague-Levy), and it is known that first-order reflection principles are provable in *ZFC*. In general, Cantor mostly referred to the epistemic transcendence of the set theoretic universe as a whole instead of focussing on its positive consequences (reflection principles).

The global reflection principle in its stronger forms is essentially a *second-order* reflection postulate. So to interpret it, we have to assign a clear meaning to the second-order quantifiers. On Zermelo's potentialist picture, this seems a tall order. Perhaps what Zermelo calls 'meta-set theory' allows quantification over absolute infinities, but Zermelo never clearly explained what he meant by 'meta-set theory'.

The pluralist interpretation of second-order quantification fares better (Uzquiano 2003). It may well give us a fairly clear interpretation of the second-order quantifiers. But on this interpretation, and therefore also on the interpretation of the second order quantifiers as ranging not only over sets but also over 'inconsistent multiplicities', the motivation for *GRP* becomes opaque. It is on this interpretation hard to make sense of the motivation for *GRP* in terms of a notion of resemblance. As far as I can see, it is only in terms of the interpretation of the second-order quantifiers as ranging over parts of the universe that the intrinsic motivation of *GRP* can be articulated. For this reason I conclude that the early Cantorian view of the set theoretical universe is mathematically the most fruitful one. Theology is not conservative over mathematics.

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